THE MATHEMATICAL GAZETTE

EDITED BY
W. J. GREENSTREET, M.A.
WITH THE CO-OPERATION OF
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AND
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LONDON
G. BELL & SONS, LTD., PORTUGAL STREET, KINGSWAY, W.C. 2.
AND BOMBAY

Vol. XII., No. 174. JANUARY, 1925. 2s. 6d. Net.

CONTENTS.

The Mathematical Association.

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The Mathematical Association.

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MATHEMATICS AND ETERNITY.

BY MISS HILDA P. HUDSON, O.B.E., Sc.D.

To all of us who hold the Christian belief that God is truth, anything that is true is a fact about God, and mathematics is a branch of theology. The relationships of men to God and to the universe have each an accurate, and even a numerical, side, capable of scientific treatment, called pure and applied mathematics. It is more than an accident that theology, in the narrower sense, continually uses mathematical terms and illustrations, some of which we shall consider. Since truth is God, and therefore one, its different departments are related and overlapping; each is capable of illuminating the others, and the lower aspects are continually recognised as symbols and sacraments of the higher.

Thus research is a mystical and a passionate quest. All scientists know this, though most have an unnecessary and unscientific shyness of admitting it. The lower passions may blind the eyes and cloud the intellect; but the high passion for truth is enlightening and steadying, and in some form or other is the only power that enables men to scorn delights and live laborious days in the pursuit of knowledge.

It is worth while for once to turn our minds away from the professional and temporal aspects of mathematics, to its relation to the things that are unseen and eternal.

Let us begin by making an act of thanksgiving for the common blessings of our intellectual life, as we give thanks for air and light, food and shelter. Just as the universe corresponds to our physical needs, and makes the life of the body possible and good and glorious, so does it also correspond to our intellectual needs, supplying us with the laws of thought: logic and arithmetic and geometry, which we use and depend on every moment. Try to imagine a world without the multiplication table, in which twice two were three to-day and five to-morrow. There would be no possibility of common sense or foresight or reason. Intellectual life would cease, we should all be reduced to idiocy, to mental death, just as without air or food we should all be reduced to physical death. Let us give thanks for the common blessings of our mental environment, which makes thought possible and good and glorious.

The phrase *mental environment* makes the great assumption that truth is something outside of, and independent of, ourselves, the same for you as for me: this underlies all study, all teaching and all religion. An old Greek, a

French child and a self-taught Indian, each finds for himself the same theory of geometrical conics. The simplest, and therefore the most scientific way of describing this, is that they have each discovered, not created, a geometry that exists by itself eternally, the same for them all, the same for teacher as for taught, the same for man as for God.

The truth that is the same for man as for God is pure mathematics as distinct from applied. The latter is also full of the divine, and the more so, the more truly scientific it is. Astronomy, not mere star-gazing, moved the Psalmist to worship: "when I *consider* Thy heavens . . ." Tennyson says the same of botany:

> "Little flower—but *if* I could *understand*
> What you are, root and all, and all in all,
> I should know what God and man is."

But however we think of heaven, it is hard to imagine astronomy and botany surviving as they are, and having much interest or importance there; for they deal with the behaviour of matter in terrestrial conditions, and if there is matter in heaven, its condition will be celestial, not terrestrial. On the other hand, it is just as hard to imagine pure mathematics not surviving. The laws of thought, and especially of number, must hold good in heaven, whether it is a place or a state of mind; for they are independent of any particular sphere of existence, essential to Being itself, to God's being as well as ours, laws of His mind before we learned them. The multiplication table will hold good in heaven, and on the question of whether twice two make four, we have the mind of Christ.

We have His mind, not as in a glass darkly, but accurately. In applied science, and particularly in the sciences of ethics and conduct, we feel after the divine truth, and reach it partially, doubtfully, inaccurately. But the thoughts of pure mathematics are true, not approximate or doubtful; they may not be the most interesting or important of God's thoughts, but they are the only ones that we know exactly.

Therefore analysis and geometry are the most sacred of all sacred studies; and they are indeed profane persons who pursue them only for examination purposes.

Because pure mathematics is thus a direct contact with God, it is possible for a Christian to dedicate his whole time to research in it: the usual gibe about its uselessness has no meaning. Other professions, and the arts and crafts of everyday life, are worth pursuing only because of their useful applications; they are indirect approaches to God, through the service of man. But mathematics is a direct approach, and can bring the sense of divine companionship more intensely than anything else; for it is God's thoughts that we are thinking after Him, and with Him. We can practise the presence of God in an algebra class, better than in Brother Lawrence's kitchen; and in the utter loneliness of an unfashionable corner of research work, better than on a mountain top. If the whole duty of man is to glorify God and enjoy Him for ever, who is in a better way of doing his duty than the mathematician?

The great idea of enjoying God points to the unity of beauty, truth and goodness; the common distinction does not go very deep: these three are one. For scientists especially, who have developed the habit of association of ideas, any suggestion of falsehood prevents pleasure and destroys beauty. It was a perverted old cynic who said that "a mixture of a lie doth ever adde pleasure."

And there is a kind of beauty found in intellectual truth and nowhere else: austere perhaps, but all the more beautiful for that. If a flower or a rainbow moves us to wonder and worship, how much more some exquisite piece of deduction, or a great theorem spanning whole ranges of applications, like Laplace's equation, or the law of the inverse square? God shines through His works as clearly in logic as in matter.

There is yet to be written a companion volume to Mrs. Gatty's *Parables from Nature*, of parables from mathematics. The convergence of infinite series makes a regular Pilgrim's Progress. The whole series is a man's life, and the terms are just terms, or years or any other periods. The first few tell you very little about him, he is still a child; as the terms pass by, the law of the series becomes apparent, and character is formed. Sooner or later he has to face the critical event of his career, to pass his convergency test, and according to how he emerges from that, so is his fate when, like all flesh, he passes to the limit. For only three score and ten or so of the terms are ever actually written down, there is no escaping the great leap to infinity. And then, if he is convergent, he reaches the sum towards which Providence has all along been guiding him, by a direct or oscillating course; the remainder term, which is the amount by which he falls short of that perfection, his sin in fact, vanishes and is blotted out. But for the divergent fellow, who strays further and further away from any appointed goal, there is the piling up of the remainder, the fearful looking for the wrath to come, and no final resting-place. And, since the condition of convergence need not hold till after any finite number of terms, there is always the possibility of a deathbed repentance.

The whole of geometry is so filled with the glory of God, that one does not know where to begin to speak of it. The projection of an infinite parabola into a finite ellipse is a very obvious parable of the Incarnation. Or take two methods of transforming a singular surface into one of normal type. The first employs a system of elementary transformations, a daily round of discipline, by which the most complicated singularities are gradually smoothed out; the other uses a single and complete operation, when many powers are brought to bear all at once, an instantaneous conversion to a final state of perfection. Either way, the surface is at last presented in its ideal form, having no spot nor wrinkle nor any such thing, changed from glory into glory, yet retaining its identity throughout.

Take that for what it is worth: no more and no less than any other parable of equal crudity. For this purpose, mathematics is on just the same footing as nature study.

But often the parallel is far more than a parable, it is a true symbol: for the sacramental principle in common life is true of mind as well as of matter. It is significant that theologians use so many mathematical terms, and they would gain enormously if they accepted, or at least understood, the mathematical definitions. For example, in theology, *infinite* has no one accepted meaning, and this vagueness brings the danger of falsehood, which is blasphemy. All mathematicians have accepted one meaning for the word, which would serve the theologians admirably, if only they knew it. An infinite aggregate is one that is equal to one of its parts, and by *equal* we mean here the usual 1, 1 relationship. No finite whole is equal to any of its parts: that is the correct form of Euclid's axiom, which is the definition of finiteness. Juliet says of her love to Romeo:

> "The more I give to thee
> The more I have, for both are infinite."

That is not poetic licence, but pure mathematics.

If this use of *finite* and *infinite* were accepted in theology we should hear no more of the "finite human mind." A finite mind could never form the idea of infinity. That the range of human thought is infinite follows directly from the definition, coupled with the fact that we can think about each of our thoughts; for thoughts about thoughts are a part of the whole set of thoughts, in 1, 1 relation to the whole set.

When a part is equal to the whole, it can enter into the same relationships with other aggregates: the whole is mapped on, or represented by, or revealed by the part. The assumption that God is mathematically infinite implies a self-revelation through a part of Himself. If God is infinite, the incarnation

is possible: if we know that He is infinite, the incarnation has happened. This is expressed in the Athanasian Creed, which calls our Lord

"Equal to the Father, as touching His Godhead,
And inferior to the Father, as touching His Manhood,"

where the statement, in order to have meaning, must refer to such aspects of the Deity as are measurable.

Again, take the word *equal*. The first time I ever spoke in public was in opening a school debate that "all men are born equal": the phrase has hardly any meaning for me now. The definition of equality can be put in many equivalent ways: take two, one algebraic, the other geometrical. If $x=5$, then x can be replaced by 5, and everything holds as before. But can Smith = Jones, in the sense that if you put Jones in Smith's place, everything is as before? ask Mrs. Smith! One of the worst features in excessive discipline, whether military or industrial, is the tendency to regard men as interchangeable units.

The second way of defining equality is Euclid's, by superposition. This is just as inapplicable to human beings, for it implies some unit of humanity, of which a whole or fractional number makes up each person: it is another form of the idea of interchangeable units. It is natural to speak of measuring one thing *against* another: this idea of equality is close to hostility, the fear lest another should be greater than I, in the only measurable things, which are not the most important. Equality is a wrong relationship on a wrong plane.

It does not help if we shift the ground, as is often done, from equality of persons to that of opportunities: for opportunity has no meaning except in relation to a particular person. For a mathematician, it may mean three years at Cambridge: for a doctor, five years in London; is then $3=5$, or London equal to Cambridge?

In the New Testament the word only occurs once as a relation between human beings, in the parable of the labourers: "Thou hast made them equal unto us, which have borne the burden and heat of the day." The passage surely contains a rebuke, not only to the bargaining spirit, but also to the inaccurate use of words. Our Lord Himself, though speaking the language of His day, uses it correctly. He never says that men are equal, but that they are brothers. Even in measurable things, equality is not always the highest ideal, and may be its denial, as in the judgment of Solomon. Brotherhood may require that to him that hath shall be given: no bread may be better than half a loaf, if the man who can use it best gets the whole.

Take another example where mathematical ideas and language help theology. Can our Lord's command to be perfect be fulfilled or no? Are some people already perfect, with nothing further to rise to, or is the command a mockery, impossible to obey? An awkward dilemma: but put into mathematical terms, the question is whether a certain limit is actually attained, whether the aggregate of human beings, ordered in a certain way, is closed or open. It is a reasonable and consistent view that our Lord is Himself the limit, and that the aggregate is closed or open according as He is or is not included as very Man. But how could that be explained to the unmathematical members of the study circle?

The two main divisions of pure mathematics, analysis and geometry, correspond with some exactness to the two great mysteries of the Christian faith, the Trinity and the Incarnation. The former is expressed in terms of the rudiments of arithmetic, one, two, three, and just as these are implied by any logical thought about the world, so the Trinity is implied by any logical thought about God.

We must start with some reasonable assumption about God. The great Christian basis, that God is love, is enough; but less would suffice, if it implied that He is one, and so has at least to that extent a numerical characteristic, a

mathematical side. If He lacked this completely, how could a mathematician worship so uninteresting a Deity ?

But if that God is one were all that could be said of His numerical aspect, He would be easily beaten, on the score of satisfactoriness and worshipfulness, by any young couple, any pair of friends or even of enemies, any *two* persons who, being different, can enter into a personal relationship with each other. If He is to hold our allegiance, He must be many without ceasing to be one : fellowship is of the essence of a Divine being. And since the Absolute is thus plural, He is at least three. For two is a poor thing that cannot exist by itself, absolutely ; it is merely a stepping stone to three, like the square of the chess-board that Alice travelled through by train. One and one make two only with the condition that they are different : otherwise they make one ; and their difference must be in respect to some third thing. Two created things may differ in place, or size, or colour ; that is, in their relation to some origin of coordinates, or scale of measurement, or spectrum, existing in the same universe. The two of the Absolute differ in their relation to a third entity as absolute as they, implied by their existence, proceeding from the one and from the other : since God is many, He is three in one. The mind cannot logically rest in the idea of two : it is forced back to one or on to three ; but there it can rest with satisfaction. For three contains within itself all the other numbers. It is seven at once : three units, three pairs and a trio ; and it carries us on to infinity, for it contains the principle of addition, of growth, of $+1$, which is so much more than 1. There is no need for God to be more than three, any larger number would be redundant.

Professor Schwarz used to begin his lectures on differential calculus by saying that before God created the world, He had to learn mathematics ; so first He created the numbers. Then the devil came along and begged for one of them, but was given nothing : so he carried off zero in great glee. That is why it is easy to prove $1=2$ by dividing both sides of equation by 0. But the professor was wrong in one point : God did not create the numbers, which are as old as Himself, part of His very nature. Thus the doctrine of the Trinity makes Christianity more acceptable, to a mathematician, than Unitarianism.

There is no denying that the pattern of the universe is based on three, from the three dimensions of apparent space, to pa, ma and baby. The very idea of dimensions only begins to be possible to beings who are conscious of three of them. Flatlanders would not have a notion even of their own two : for the underlying idea is difference of direction, and rotation, and behind this, both physically and geometrically, is the idea of an axis of rotation perpendicular to the plane, requiring a third dimension for its existence. We space-dwellers, from our superior eminence, can dissociate the ideas, and deal with two dimensions alone just as we can deal with four. But before his excursion into space, the Square could not have had the idea of dimensions at all : for again, whosoever hath, to him shall be given ; and whosoever hath not, from him shall be taken even that which he seemeth to have.

But though pure mathematics is thus full of suggestions and parallels and confirmations of these high doctrines, it never asserts them : that belongs to theology proper. It only asserts that if a mathematician holds that God has certain properties, then he must also hold such articles of the catholic faith as can be deduced from the assumptions. It only attains to the certainty, which is its particular glory, at the cost of refusing to make any assertions about the universe, but only about the consistency of assertions with each other. *If ABC* is an equilateral triangle, Euclid will tell us all sorts of entrancing things about it : he never says that three material points do form an equilateral triangle ; that belongs to surveying, not geometry.

In applied science, on the other hand, we have theories asserted as true ; in the nature of the case, this is an entirely different meaning of truth. No confusion need arise, for the pure meaning has no significance here. Applied

truth is capable of gradations. A theory is more or less true according as it is a more or less satisfactory description of certain observed facts. This satisfactoriness is aesthetic as well as logical; its chief element is simplicity, which is a form of beauty.

The heliocentric theory of the solar system did not account for more facts than the "Eccentricks and Epicycles, and such Engines of Orbs" of the earlier astronomers, which were carried to the degree of complication needed to describe the observed facts of the time. But this caused such a loss of simplicity, compared with the earlier theory of seven celestial spheres, so intense a dissatisfaction and strain, that the new idea was able to burst through into the mind of Copernicus: or rather an idea, old as the heavens themselves, came to its right place at last. It prevailed by being, not a fuller account than the epicycles, but a simpler, not so much through its truth as through its beauty, proved in the only way possible for a physical theory, by satisfying men's souls as well as their minds.

We owe a great debt to the old school, cranks and faddists though they were, Their courage and persistence, in stretching the current theories to all the facts. alone gave the needed background and stimulus for the next advance. It is often only the cranks who have just this kind of courage.

As in astronomy, so in politics. For centuries we have fitted the facts of everyday intercourse, between nations and neighbours, to a theory based on the old geocentric idea of division—race, class, sex, denomination, education. This has been carried through to the intolerable complication and strain of the war and the peace. On every side there is abroad the new idea, old as God Himself, based on unity instead of division. The two sets of people who ought to be the first to accept the new basis of human relationships are on the one hand scientists, whose work is already based on the unity of truth, and on the other hand Christians, who believe in the fatherhood of God; and before all others, Christians who are scientists. We can pass naturally from abstract mathematical research to practical work for European Student Relief or the League of Nations, with a full sense of continuity and appropriateness. The tragedy of our times is that so many people, partly from fear of being thought cranks, are wavering between the two contradictory principles, between sacrifice and security, love and fear. There is neither simplicity nor satisfaction for such, till they take a more scientific attitude. But wherever the principle of love is applied consistently, the simplification which it brings to all the relationships of life carries its own conviction.

In spite of the clear distinction between the processes of pure and applied mathematics, they have more in common than appears at first sight. In any argument, the last step of all is not thought, but illumination. I go through a proof, and at first reading do not see it; I go through it again, the exposition is the same, my mental capacity has not grown conspicuously in a few minutes, yet there is all the difference, I see it this time. Psychologists and educational experts have their own way of describing what happens; but we can also say that a vision has dawned, of God as embodied in that particular theorem, that the Holy Spirit has led me that much farther into all the truth, the spirit within has leaped to meet the divine truth without, that corresponds to it and compels it.

It may well be that in eternity this leap of recognition will be all that remains of the reasoning process; the least of us will contemplate the whole of mathematics and see that it is an aspect of God by then grown familiar,

> "Non dimostrato, ma fia per sè noto,
> A guisa del ver primo, che l' uom crede."

H. P. Hudson.

THE POSTULATE OF PARALLELS.

By M. J. M. Hill, Sc.D., F.R.S.

1. There are in the *Mathematical Gazette* for October 1924, three communications on the above subject.

It will make my remarks on these clearer if I refer to the contents of my paper on the Postulate of Parallels in the *Gazette* for December 1923, and make an addition thereto. In that paper the demonstrations of Euc. I. 27 and 29 are deduced from Wallis's *Postulate of Similarity*, viz.: "To every figure there exists a similar figure of arbitrary magnitude" taken as a *substitute* for Euclid's Postulate of Parallels. The demonstration of Euc. I. 27 in this manner was first given (I believe) by Professor Nunn in the *Gazette* of May 1922. He has not, however, published in full his demonstration of Euc. I. 29. He was so kind as to communicate it to me, but I hope he will some day publish it in full. It is entirely different from the demonstration of Euc. I. 29 which I published in the *Gazette* of December 1923, which involves three successive applications of the Postulate of Similarity and makes use of Euc. I. 27 as proved by Professor Nunn, but does not include the proof of any other proposition.

2. To the contents of that paper I am now able to add demonstrations of Euc. I. 16 and 17 by the aid of the Postulate of Similarity.

Euc. I. 16.

Let ABC be a triangle. Produce BC to any point D. Then draw CE on the same side of BCD as A is, so as to make the angle DCE equal to the angle ABC. Then by the proof of Euc. 1. 27 as given * in the *Gazette*, May 1922, or December, 1923, it is known that CE cannot meet BA. Therefore CE cannot lie in the angle BCA. If it did it would by Pasch's Axiom, meet BA between B and A.

Therefore CE lies in the angle ACD as shown in the figure.

$$\therefore\ A\hat{C}D > E\hat{B}C,$$

$$\text{but } E\hat{C}D = A\hat{B}C;$$

$$\therefore\ A\hat{C}D > E\hat{C}D,$$

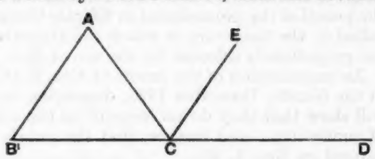

Fig. 1.

i.e. the exterior angle of the triangle ABC is greater than the interior and opposite angle. This is Euc. I. 16.

Euc. I. 17.

Further, $A\hat{B}C + A\hat{C}B = E\hat{C}D + A\hat{C}B$, which is less than two right angles. This is Euc. I. 17.

Thus the group of propositions Euc. I. 16, 17, 27, 28 (substantially the same as 27) and 29 may be deduced from the Postulate of Similarity.

3. The definition of similar figures which Wallis had in mind in stating his Postulate was doubtless Euclid's, which may be stated shortly thus:

(i) Corresponding angles of two similar figures are equal.

(ii) Corresponding lengths of two similar figures have the same ratio.

It is important to observe that in using Wallis's Postulate to prove Euc. I. 16, 17, 27, 28 and 29, only the first part of the definition of similarity has been required.

For it has been assumed only that if ABC be a triangle known to exist, and if $B'C'$ be any line of arbitrary length, and if the angle $C'B'X$ be made

* Euclid's proof involves the infinity of the straight line.

equal to the angle CBA, and if the angle $B'C'Y$ be made equal to the angle BCA in the same plane as $C'B'X$, and on the same side of $B'C'$ as X, then the Postulate of Similarity requires that $B'X$ and $C'Y$ should meet in some point A', and that the angle $B'A'C'$ should be equal to the angle BAC.

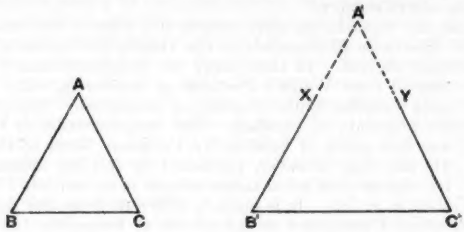

Fig. 2.

It is to be particularly noticed that it is not necessary to introduce the concept of ratio into the argument, a procedure which would result in making the reasoning more complicated.*

And further the above demonstration shows the relation of Euc. I. 16 to the Postulate of Similarity or its equivalent the Postulate of Parallels, viz. :

Euc. I. 16 is a consequence of, not a preliminary to, the Postulate of Parallels.

When, as in Elliptic Geometry the Postulate of Parallels does not hold, it is not true for triangles of all sizes that the external angle of a triangle is greater than the interior and opposite angle.

Students of the work of Saccheri and Lambert will remember the notes in Engel and Stäckel's *Theorie der Parallellinien*, in which it is pointed out that the proofs of the propositions in Elliptic Geometry (or, as Saccheri and Lambert called it, the Geometry in which the Hypothesis of the Obtuse Angle is valid) are prejudicially affected by the use of Euc. I. 16.

An examination of the proofs of Euc. I. 16, 17, 27, 28 and 29 given here and in the *Gazette*, December 1923, depending on Wallis's Postulate of Similarity will show that they do not depend on the concept of ratio or on the principle of continuity ; and further, that the proofs of Euc. I. 27, 28 and 29 do not depend on Euc. I. 16.

I can now go on to the three communications in the October number of the *Gazette* bearing on the subject.

(A) The method proposed by Mr. Krishnaswami Ayyangar in the *Gazette* makes use of the concept of ratio on p. 191, and of Euc. I. 16 on page 192. It is, therefore, more complicated than the proofs given in the *Gazette*, December 1923. Apart from this there is some interesting work on pp. 192, 193.

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* We may go further and show that what has been described above as the second part of the definition of similar figures can be deduced as a conclusion from the first part if we take account of the postulate of parallels (or similarity) and the concept of ratio. For if A, B, C be the points of the first figure corresponding to A', B', C' respectively of the second figure, then by the first part of the definition of similar figures the triangles ABC, $A'B'C'$ have the angles of the one respectively equal to those of the other. Then making use of the postulate of parallels and the concept of ratio, we may obtain the result given in Euc. VI. 4, that

$$(BC : B'C') = (CA \,.\, C'A') = (AB : A'B').$$

Now let the points P, Q, R, S of the one figure correspond respectively to P', Q', R', S' of the other figure. To prove the second part of the definition of similar figures, it is enough to show that

$$(PQ : P'Q') = (RS : R'S').$$

From the similarity of the triangles PQR, $P'Q'R'$, we get

$$(PQ : P'Q') = (QR : Q'R') = (RP : R'P').$$

From the similarity of the triangles RPS, $R'P'S'$, we get

$$(RP : R'P') = (PS : P'S') = (RS : R'S').$$

Consequently $$(PQ : P'Q') = (RS : R'S').$$

(*B*) The article on Parallelism and Similarity (pp. 195-201) by Mr. D. K. Picken does not make use of the Postulate of Similarity, and does not therefore fall strictly within the scope of this article. It may, however, be remarked that it makes the argument leading up to the explanation of the Postulate of Parallels depend on Euc. I. 16 as do Euclid, Saccheri and Lambert. To this matter I have referred above.

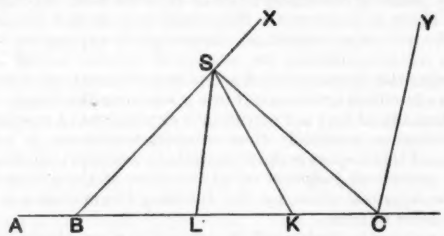
The fifth section of Mr. Picken's paper (pp. 197-198), in which he gives the essentials of an elementary treatment of similarity, is in a form which I have not seen in print before. I had some experience in the use of the figure on p. 197, during the last two or three years of my tenure of office at University College. I think it is probably the best way of introducing the subject to beginners.

(*C*) Mr. Piggott's reply (pp. 203-4) to my criticism of a point on p. 39 of the *Report on the Teaching of Geometry*, commences with a reference to a well-known algebraic theorem, which he states thus:

"If $k>f(a)$ and $<f(b)$ the function $f(x)$ being finite, continuous and single-valued between a and b, then there exists a value of x, between a and b for which $f(x)=k$."

But surely the conclusion of the theorem should be "there exist one or more values of x between a and b for which $f(x)=k$." (See Lamb's *Infinitesimal Calculus*, p. 17, the last three lines: "In other words, a continuous function cannot pass from one value to another without assuming once (at least) every intermediate value.")

In the light of this last statement consider the argument at the foot of page 203, in the case in which there are two points K and L between B and C, such that $B\hat{L}S=B\hat{K}S=A\hat{C}Y$, a case which actually occurs when the plane is elliptic. It would then be necessary to use the Postulate of Similarity to rule this case out on the ground that there cannot exist two triangles, viz.: BLS



and BKS with the angles at B and L respectively equal to those at B and K, whilst the angles at S are unequal.

What would the beginner think of this? Would he not, remembering Euc. I. 16, think that $A\hat{B}S>B\hat{L}S>B\hat{K}S>B\hat{C}S$, and therefore that it was impossible to have $B\hat{L}S=B\hat{K}S$. It seems to me that he would be inclined to argue that, as a point P moved from C to B, the angle $B\hat{P}S$ would continually increase.

Is it advisable at this stage of his knowledge to bring him up against the alternatives in the geometrical analogue of the above mentioned algebraic theorem?

I would venture to urge as strongly as I can that an adherence to the line of argument recommended on p. 39 of the *Report on the Teaching of Geometry* in the matter which I have criticised will effectually wreck the prospects of a most important reform.

M. J. M. HILL.

THE TEACHING OF GEOMETRY IN SCHOOLS.

A REPORT PREPARED FOR THE MATHEMATICAL ASSOCIATION.

By C. V. Durell, M.A.

There is little doubt that many teachers are seriously bewildered by this Report. It is not an easy document to read and frequently it is difficult to ascertain the precise nature and application of its recommendations. It suffers from attempting to cover too much ground: the reader is confused by the double purpose it tries to serve, (i) instruction on the axiomatic basis of geometry for teachers who have made no special study of the logical foundations of the subject, (ii) pronouncements on practical procedure in teaching and geometrical method.

No one can doubt the importance of the first of these two objects and numerous teachers will welcome gratefully an intelligible account of the scrutiny modern research has made of the foundations of geometry. But such an account is not a matter of opinion or experience, but of scientific fact. It should be issued as a *tract*,* not as a *report*. What modifications should be made in elementary teaching in the light of modern research must, however, be determined by the school teacher. The reports of the Association have in the past been confined to methods of teaching, syllabus, etc., and their influence has been due to the fact that they have summarised teaching experience: they have not hitherto been used to provoke discussion, but to urge particular methods which have been tested by experience.

This distinction between a tract and a report is not trivial for the following reasons:

(i) A tract is the work of an expert: a committee may suggest minor improvements by pointing out ambiguities of expression or difficulties the ordinary reader is likely to encounter. But criticism as to fact does not arise.

A report is the work of a committee: it attempts to express the best informed experience, its recommendations are matters of opinion, not of fact.

(ii) The smaller the limits to which a report is confined, the more effectively a committee can handle it and mould it into a workmanlike shape. If, however, numerous statements of fact are woven into expressions of opinion, the document may become so unwieldy that committee-criticism is impracticable. The dimensions of this Report make it possible to doubt (*a*) whether the Report represents the *considered judgment* of all members of the sub-committee and (*b*) whether the approval given by the Teaching Committee was much more than a mere matter of form.

(iii) When a method is condemned, it may not be clear whether the objection is based on grounds of logic (the criticism of the expert) or experience (the criticism of the practical teacher).

(iv) The qualifications for presenting an authoritative scientific statement such as a tract must contain are not necessarily associated with the elementary teaching experience essential for an authoritative pronouncement on methods of procedure and without which it is difficult to know the kind of practical details for which the ordinary teacher will look. In the present case it is certain that a great many teachers will be left in doubt as to what practical procedure the committee recommends and what precisely are the importants changes they are asked to make. There is a real danger that geometrical teaching may be injured by a misunderstanding and mis-application of the views expressed.

If these reasons carry conviction, it appears that much of the matter of the Report is out of place, valuable though it is. The following remarks refer only to those sections which appear to make recommendations affecting the practical teacher.

* Cf. "Cambridge Tracts in Mathematics."

There are two main themes in the Report, (*a*) Congruence, (*b*) Parallelism. In both cases it is not always clear whether the Report is calling attention to an ideally scientific method or to a method considered feasible for class-room use.

(*a*) Congruence.

The "Principle of Congruence" as set out in the Report will strike terror into many teachers' hearts. Compare it with the concrete practical method of the Board of Education's circular. Fundamentally they agree. In the circular the pupil is asked to say what measurements must be made in order to copy a given triangle. The Report quotes this with approval, but sets out in a formal statement the underlying principle (pp. 29-31). Is this simply instruction for the teacher? Probably not, for the Report appears to advise (pp. 30, 31) that the statement of the principle, analysed into axioms, and the deduction from it of the formal proofs of I. 4 and I. 26 should be taught to pupils of ages 15-17. Such advice may do grave harm. It calls for exceptional powers of exposition from the teacher and a high order of intelligence on the part of the pupil. In the absence of either, this method would be disastrous. Moreover the subtleties of language involved (cf. the distinction between constructing a figure and instructions that direct attention to a figure, p. 32) are beyond the capacity of all ordinary pupils.

By all means let superposition proofs be cut out from every syllabus and every examination. But may we not then adopt the method of the circular for obtaining the results of I. 4 and I. 26 and thereafter treat these as part of our present axiomatic basis? To substitute the proofs of this Report will condemn most boys to committing to memory a series of phrases and sentences to which they attach no meaning.

(*b*) Parallelism and Similarity.

It is generally accepted that the concept of similarity should be introduced early: this view is endorsed by the Report. But the controversial paragraphs deal with the application of the "Principle of Similarity" to the organising stage (ages 15-17).

A previous article in the *Gazette* has criticised one of the suggested proofs. But apart from any question of vigour, the real obstacle is the intrinsic difficulty of the proposed method. It is unlikely that many attempts will be made to follow it out unless a specially written text-book is provided. It therefore seems needless to examine the reasons given in support of the recommendation to substitute the "Principle of Similarity" for Euclid's method or one of the modern equivalents. The report then proceeds to comment on the use of the idea of direction in the theory of parallels. It is not easy to discover precisely what these comments mean. No one will dispute the statement that a logical proof of the fundamental parallel theorems cannot be based on the idea of direction or the idea of rotation. Nor does the Board's circular say it can be. It does say that these two ideas can profitably be used to make the pupil understand what these theorems mean and to link up his everyday experience with his geometrical notions. If our axiomatic basis is to be so broad as to include the fundamental parallel theorems, this seems the most practicable way of approaching the subject. The danger is that this specific criticism of the Board's circular may lead some teachers to think that the use of such illustrations in building up the axiomatic basis is condemned by the Report.

It is, of course, desirable to consider how far (if at all) an attempt should be made in the *organising stage* to narrow the axiomatic basis. This is discussed on page 20, etc. [What by the way is the meaning of the statement, p. 20: "the first group of theorems (angles at a point) should certainly be proved here"? Can anything more be said at this stage than has been said long ago in any geometrical course? What sort of formal proof is wanted?]

The Report recommends that the axiomatic basis should be narrowed as follows: (i) the deductions of I. 4 and I. 26, from the Principle of Congruence,

(ii) the treatment of parallelism and similarity *either* by Euclid *or* by the Principle of Similarity.

This problem of the narrowing of the basis is of fundamental importance and demands a wide interchange of opinion among teachers: the definite recommendation of the Report seems premature. It cannot be denied that the enumeration or exposition of all the axioms required for a scientific study of geometry is specialist work: school geometry must be built on a broader basis than will satisfy the philosopher. Compromise is necessary and it rests with teachers to discover and decide what compromise will give the best results. All our axioms must clearly be in accord with a boy's experience and it is equally obvious that proofs must be given as soon as results occur which are not self-evident. But subject to these two limitations, the foundation should be determined by what experience shows is the best jumping off point for school work and best adapted for extensive reading and applications. The more time is spent on narrowing the basis, the less time remains for developments. This at any rate is the question at issue, and it is doubtful whether the time is yet ripe for a dogmatic pronouncement.

The Report deals with many other matters of great interest, too numerous to discuss within the bounds of a single article. The writer however, disagrees so seriously with the section on the treatment of limits that he hopes to be allowed to express his views on this subject on a later occasion.

C. V. DURELL.

GLEANINGS FAR AND NEAR.

277. Relativity.

Einstein's theory is very simple to state,
" The foundation of Truth lies in Relativity,"
Light is bent ?, we thought it straight,
May be Laziness is then really Activity.
What we accepted as straight, is crooked ?,
Is the right we hold then proved wrong ?
And wrong proved relatively right ? Who could
Be certain the weak are not the strong ?
You might think that we won the war,
But the Great Scientist knows full well,
By relative reasonings as truth to tell,
That what's won is really lost for sure,
And what's lost is won, what's more,
Your Heaven upon Earth would be Hell.

—John Watson. *The Westminster Gazette.*

278. Charles Hutton was appointed, by public examination, Professor of Mathematics in the Royal Military Academy of Woolwich. " This situation had been held by some mathematicians of the first eminence. The celebrated Thos. Simpson had formerly occupied the chair, and fell a sacrifice to the low and unhealthy situation in which the academy at that time was placed. Mr. J. L. Cowley, who succeeded, found, after he had been there nearly as long as Simpson had been, that his health was declining, and consequently he resigned.—J. Bruce, *A Memoir of Charles Hutton*, LL.D., F.R.S. (1823). [Cowley's *Perspective*, 1766, is in the Library.]

279. When Wollaston " was nearly in the last agonies, one of his friends having observed, loud enough for him to hear, that he was not conscious of what was passing around him, he made a sign for pencil and paper. He then wrote down some figures, and after casting up the sum, returned the paper. The amount was correct."—Sir John Barrow, *Sketches of the Royal Society*, p. 65. [We have elsewhere read that he wrote down an Arithmetical Progression !]

SOME THEOREMS IN GEOMETRY AND SOME SUGGESTIONS.

By R. T. Robinson, M.A.

In a system in which we have given a circle and the relations between it and certain points and lines, certain changes take place in these relations in another system in which a conic takes the place of the circle. To enable these changes to be more easily seen and interpreted, the writer has found it very convenient to use certain modes of expression. For example:

(1) The st. lines SA, SB are $\perp^r$ if they are harmonically conjugate to the st. lines joining S to the circular points at infinity. If these st. lines SA, SB are harmonically conjugate to the st. lines joining S to the real base points ω_1, ω_2, *i.e.* if $S(A\omega_1 B\omega_2) = -1$, it is a great convenience to express the fact in this way: SA is $\perp^r(\omega_1\omega_2)$ to SB.

(2) If the st. line joining $\omega_1\omega_2$ meets AB in B and if $A(S\omega_1 B\omega_2) = -1$, this will be expressed by the statement: AS or SA is $\perp^r(\omega_1\omega_2)$ to AB.

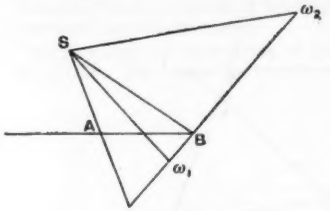


Fig. 1.

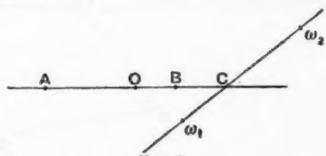


Fig. 2.

(3) If $\omega_1\omega_2$ meets AB in C and if $(AOBC) = -1$, then O is the mid-point $(\omega_1\omega_2)$ of AB.

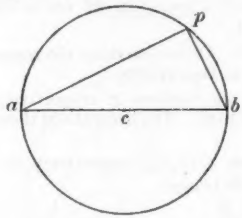


Fig. 3.

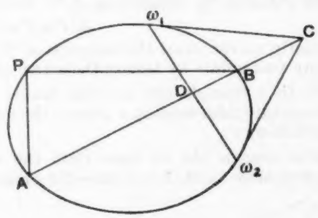


Fig. 4.

(4) In a circle, when ab passes through the centre c pa is $\perp^r$ to pb.

and the locus of a point p which moves so that the st. lines pa, pb joining it to two fixed points a and b are $\perp^r$ is a circle and c is the mid-point of ab.

In a conic when AB passes through C the pole of $\omega_1\omega_2$

$$PA \text{ is } \perp^r (\omega_1\omega_2) \text{ to } PB$$

i.e. $$P(A\omega_1 B\omega_2) = -1,$$

for $$P(A\omega_1 B\omega_2) = \omega_1(A\omega_1 B\omega_2) = (ACBD) = (ADBC) = -1$$

and the locus of a point P which moves so that the st. lines PA, PB joining it to two fixed points A and B are $\perp^r$ $(\omega_1\omega_2)$ is a conic passing through $AB\omega_1\omega_2$ and C is the mid-point $(\omega_1\omega_2)$ of AB.

(5) This mode of expression is useful in constructions. For example, to find the centre of a circle when 3 points ABC on the circle are given, we bisect AB and AC at right angles and the point of intersection of the $\perp^{rs}$ is the centre.

If 5 points on a conic $ABC\omega_1\omega_2$ are given and we want to find the pole of $\omega_1\omega_2$, we proceed in a similar way. We first bisect $(\omega_1\omega_2)$ the st. lines AB and BC, *i.e.* we find points E and G such that

$$(BEAD) = -1 \quad \text{and} \quad (BGCF) = -1.$$

We then draw $EH \perp^r (\omega_1\omega_2)$ to AB, *i.e.* so that $E(D\omega_1 H\omega_2) = -1$ and $GK \perp^r (\omega_1\omega_2)$ to BC, *i.e.* so that $G(F\omega_2 K\omega_1) = -1$. T, the point of intersection of EH and GH, is the pole of $\omega_1\omega_2$. *i.e.* $T\omega_1$ and $T\omega_2$ are the tangents to the conic at $\omega_1\omega_2$.

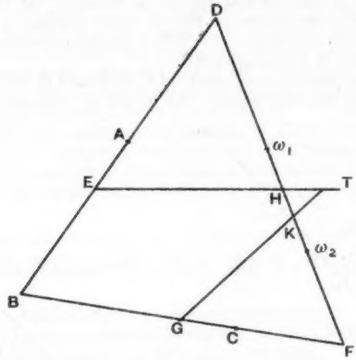


FIG. 5.

We can find the tangent at A by drawing $AP \perp^r (\omega_1\omega_2)$ to AT *i.e.* so that

$$A(P\omega_1 T\omega_2) = -1.$$

Similarly we can draw the tangents at B and C. Or we can draw the tangents at any two points by taking these two points as base points.

(6) In a triangle the st. lines drawn from the vertices $\perp^r$ respectively to the opposite sides meet in a point—the ortho-centre. The generalised theorem is as follows:

In a triangle the st. lines from the vertices $\perp^r (\omega_1\omega_2)$ respectively to the opposite sides meet in a point—the ortho-centre $(\omega_1\omega_2)$.

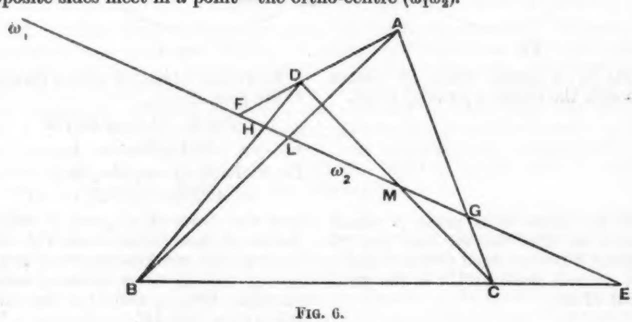


FIG. 6.

Thus, if $\quad AD$ is $\perp^r(\omega_1\omega_2)$ to BC, then $A(\omega_1 F\omega_2 E) = -1$,
and if $\quad BD$ is $\perp^r(\omega_1\omega_2)$ to CA then $B(\omega_1 H\omega_2 G) = -1$.
i.e. in the quadrilateral $ABCD$, the pair-points (F, E), (G, H) are harmonically conjugate to $\omega_1\omega_2$; $\therefore$ the pair-points L, M are harmonically conjugate to $\omega_1\omega_2$, *i.e.* $C(\omega_1 L\omega_2 M) = -1$,

i.e. CM is $\perp^r(\omega_1\omega_2)$ to AB,
or CD is $\perp^r(\omega_1\omega_2)$ to AB,

i.e. D is the ortho-centre $(\omega_1\omega_2)$ of the $\triangle ABC$.

This is another way of stating Desargues' Theorem.

The Theorem can also be expressed thus: In a quadrilateral $ABCD$ each angular point is the ortho-centre $(\omega_1\omega_2)$ of the triangle formed by the other three points.

And further, if any conic passes through $ABCD$, ω_1 and ω_2 are conjugate points with respect to that conic. Therefore, if ABC is a $\triangle$ inscribed in a conic and $\omega_1\omega_2$ are conjugate points with respect to this conic, the ortho-centre $(\omega_1\omega_2)$ of the $\triangle ABC$ lies on the conic. This is a generalisation of the Theorem that if a triangle is inscribed in a Rectangular Hyperbola, its ortho-centre also lies on the curve.

(7) The angle of intersection of two st. lines SA, SB is usually denoted as the $\angle ASB$. The writer has found it a great convenience to denote the inclina-

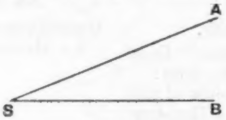
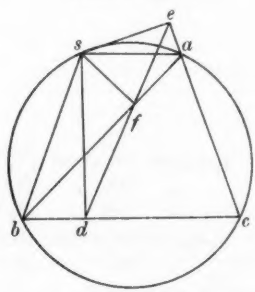
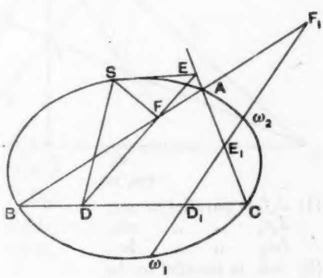


FIG. 7.

tion as $\angle S(AB)$. As examples take the generalisation of the theorems known as Simson's line and the nine-points circle.

Simson's Line.	*Generalisation of Simson's Line.*
 FIG. 8.	 FIG. 9.
	Let $\omega_1\omega_2$ meet $BC \,.\, CA \,.\, AB$ respectively in $D_1E_1F_1$.
Draw sd $\perp^r$ to bc.	Draw SD $\perp^r(\omega_1\omega_2)$ to BC, *i.e.* find D so that $S(D\omega_1 D_1\omega_2) = -1$.
Draw se $\perp^r$ to ca.	Draw SE $\perp^r(\omega_1\omega_2)$ to CA, *i.e.* find E so that $S(E\omega_2 E_1\omega_1) = -1$.
Draw sf $\perp^r$ to ab.	Draw SF $\perp^r(\omega_1\omega_2)$ to AB, *i.e.* find F so that $S(F_1\omega_2 F\omega_1) = -1$.

(1) because fs is $\perp^r$ to fa and es is $\perp^r$ to ea;

$\therefore$ $s\ e\ a\ f$ lie on a circle

and $\angle f(se) = \angle a(se)$.

(2) $sabc$ lie on a circle;

$\therefore$ $\angle a(se) = \angle b(sc)$.

(3) ds is $\perp^r$ to db and fs is $\perp^r$ to fb;

$\therefore$ $sbdf$ lie on a circle;

$\therefore$ $\angle b(sd) = \pi - \angle f(sd)$

or $\angle b(sc) = \pi - \angle f(sd)$.

(4) $\therefore$ from (2) $\angle a(se) = \pi - \angle f(sd)$ and from (1) $\angle f(se) = \pi - \angle f(sd)$;

$\therefore$ fe coincides in direction with fd.

i.e. def are collinear.

(8) *The Nine-points Circle.*

Let d, e, f, be the feet of the $\perp^{rs}$ from the angular points on to the sides:

Let $d_2e_2f_2$ be the mid-points of the sides and let kmn be the mid-points of ap, bp, cp respectively.

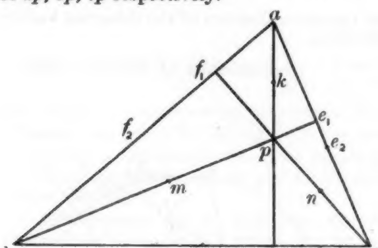


FIG. 10.

(1) d_2f_2 is parallel to ac,

d_2e_2 „ „ ab,

f_2e_2 „ „ bc

(2) nd_2 is parallel to bp,

ne_2 „ „ ap.

(3) $\angle d_2(nf_2)$ is a right angle,

$\angle e_2(nf_2)$ „ „

$\therefore$ n lies on the circle $d_2e_2f_2$ and nf_2 is a diameter.

Similarly k and m lie on this circle.

(4) $\angle f_1(nf_2)$ is a right angle; $\therefore$ f_1 lies on the circle on nf_2 as diameter, *i.e.* it lies on the circle $d_2e_2f_2$. Similarly d_1e_1 lie on this circle.

(1) because FS is $\perp^r(\omega_1\omega_2)$ to FA and ES is $\perp^r(\omega_1\omega_2)$ to EA;

$\therefore$ $SEAF\omega_1\omega_2$ lie on a conic

and $F(S\omega_1E\omega_2) = A(S\omega_1E\omega_2)$

or $A(S\omega_1C\omega_2)$.

(2) $SABC\omega_1\omega_2$ lie on a conic;

$\therefore$ $A(S\omega_1C\omega_2) = B(S\omega_1C\omega_2)$

or $A(S\omega_1E\omega_2) = B(S\omega_1C\omega_2)$.

(3) DS is $\perp^r(\omega_1\omega_2)$ to DB and FS is $\perp^r(\omega_1\omega_2)$ to FB;

$\therefore$ $SBDF\omega_1\omega_2$ lie on a conic;

$\therefore$ $B(S\omega_1D\omega_2) = F(S\omega_1D\omega_1)$

or $B(S\omega_1C\omega_2) = F(S\omega_1D\omega_2)$.

(4) $\therefore$ from (2)

$A(S\omega_1E\omega_2) = F(S\omega_1D\omega_2)$

and from (1)

$F(S\omega_1E\omega_2) = F(S\omega_1D\omega_2)$;

$\therefore$ FE coincides in direction with FD.

i.e. DEF are collinear.

Generalisation of the nine-points circle i.e. the eleven-points conic.

Let $\omega_1\omega_2$ meet the sides $BC \,.\, CA \,.\, AB$ in DEF respectively: let $D_1E_1F_1$ be the feet of the $\perp^{rs}(\omega_1\omega_2)$ from the angular points on to the sides,

i.e. $A(D\omega_1D_1\omega_2) = -1,\quad B(E\omega_2E_1\omega_1) = -1,\quad C(F_1\omega_2F\omega_1) = -1.$

Let $D_2E_2F_2$ be the mid-points $(\omega_1\omega_2)$ of the sides,

i.e. $(BD_2CD) = -1,\quad (AECE_2) = -1,\quad (BFAF_2) = -1.$

Let $R \,.\, M \,.\, N \,.$ be the mid-points $(\omega_1\omega_2)$ of $AP \,.\, BP \,.\, CP$ respectively,

i.e. $(ASPK) = -1,\quad (BMPR) = -1,\quad (CNPN_1) = -1.$

We proved in (6) above that the $\perp^{rs}(\omega_1\omega_2)$ from ABC on to the opposite sides meet in a point P the ortho-centre $(\omega_1\omega_2)$.

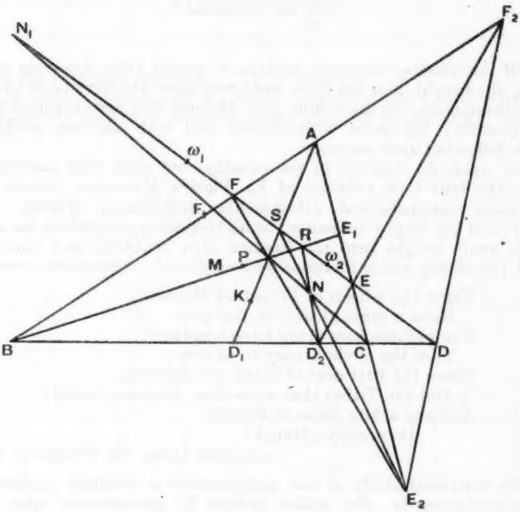


FIG. 11.

(1) D_2F_2 meets AC on $\omega_1\omega_2$; $\therefore$ D_2EF_2 are collinear,
D_2E_2 „ AB „ $\therefore$ D_2E_2F „ „
F_2E_2 „ BC „ $\therefore$ F_2E_2D „ „

(2) ND_2 meets BP on $\omega_1\omega_2$ $\therefore$ RND_2 „ „
NE_2 „ AP „ $\therefore$ SNE_2 „ „

(3) $D_2(\omega_2N\omega_1F_2) = (\omega_2R\omega_1E)$ measuring on $\omega_1\omega_2$,

$= B(\omega_2R\omega_1E)$,

$= B(\omega_2E_1\omega_1E) = -1$;

$\therefore$ D_2N is $\perp^r(\omega_1\omega_2)$ to D_2F_2.

$E_2(\omega_2N\omega_1F_2) = (\omega_2S\omega_1D)$ measuring on $\omega_1\omega_2$,

$= A(\omega_2S\omega_1D)$,

$= A(\omega_2D_1\omega_1D) = -1$;

$\therefore$ E_2N is $\perp^r(\omega_1\omega_2)$ to E_2F_2;

$\therefore$ N lies on the conic $D_2E_2F_2\omega_1\omega_2$ and NF_2 passes through the pole of $\omega_1\omega_2$.

Similarly we can prove that K and M lie on this conic.

(4) $$F_1(\omega_2N\omega_1F_2) = F_1(\omega_2C\omega_1A) = -1,$$
for F_1C is $\perp^r(\omega_1\omega_2)$ to F_1A or AB;

$\therefore$ F_1 lies on the conic through $NF_2\omega_1\omega_2$ which is such that NF_2 passes through the pole of $\omega_1\omega_2$: but D_2E_2 lie on this conic;

$\therefore$ F_1 lies on the conic $D_2E_2F_2\omega_1\omega_2$.

Similarly D_1E_1 lie on this conic;

$\therefore$ a conic will pass through $D_2E_2F_2\omega_1\omega_2D_1E_1F_1KMN$.

(To be continued.)

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280. Of Archbishop Temple's mother it is said that, knowing not a word of Latin, she taught him his Eton grammar from the first page to the last; a bad arithmetician, she took him, with the aid of a key, through the whole of Bonnycastle; the same with Euclid and with algebra, intelligence in each case following upon memory.

In later years he masters in six months—we read with something of a shudder—the four folio volumes of La Place's *Mécanique Céleste.* To his extraordinary mathematical attainments Archdeacon Wilson, a Senior Wrangler and his Rugby colleague, bears testimony, recording his analytical dexterity, swift insight into the essence of a problem, and extraordinary power of visualizing space relations and numbers. *Athenaeum, circa* 1906.

281.
There the wheels of Perpetual Motion
Make a musical whir in the air;
The philosophers there have a notion
That the circle is easy to square;
There the flatt'ners of earth are deficient,
And the Tribes that were Lost, they are found;
And the arkite ideas of Bryant
Do greatly abound!
—Andrew Lang, *The Temple of Bosh.*

282. The universal study of the mathematics is strongly marked by Mr. Nunn's advertisements, who makes clothes by geometrical rules, and has discovered a problem whereby he is enabled to cut them out with an accuracy before unknown. This, as the parts to be fitted are circumscribed by curve lines of different natures, shows his investigations must depend on the more sublime parts of geometry.—F. Grose, *The Olio,* 1796, p. 252.

283. Logicians and grammarians of the old type always insist that . . . those languages are illogical which use a double negative as a strengthened negative. Thus formulated, however, the verdict cannot be maintained. A reference to the mathematical law that $- - = +$ is not conclusive, for a linguistic negative is not the same thing as the mathematical negative sign; as we have seen, "not four" means "either more or (generally) less than four," while 4 means one definite point as much below 0 as $+4$ is above 0. Therefore $-(-4)$ necessarily comes to mean $+4$. But it is different in language.—*Logic and Grammar,* by O. Jesperson, 1924, p. 8.

284. The altercations being renewed, the engineer proceeded to the illustration of his mechanicks, tilting up his hand like a balance, thrusting it forward by way of lever, embracing the naturalist's nose like a wedge between two of his fingers, and turning it round with the momentum of a screw or peritrochium.—*Peregrine Pickle,* iv., c. 10.

A GEOMETRICAL CONSTRUCTION FOR A CARDIOID.

By H. E. Piggott, M.A.

One of the best-known examples of a caustic curve in Geometrical Optics is that obtained by reflection from the arc of a circle of rays emanating from a point on the circumference of the circle. The enveloping curve is known to be a cardioid. It has not, however, I think, been pointed out that the analysis of this problem leads to a very simple construction from which a cardioid can be rapidly and accurately drawn.

In Fig. 1, A is the point-source of light, C the centre of the circle. Rays reflected from the neighbourhood of B will pass through O, the 'image' of A by reflection in the mirrored arc, given by

$$\frac{1}{BO}+\frac{1}{BA}=\frac{2}{a}$$

where a is the radius of the circle. Thus $BO=\frac{2}{3}a$.

O will be the cusp of the caustic curve. Refer to polar co-ordinates through O. Consider the ray AP, reflected along PQ, the angles CAP, CPA, CPQ being each α.

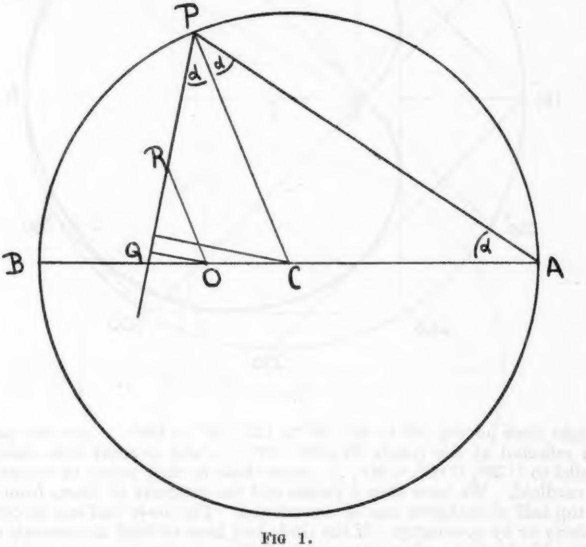

Fig 1.

The length of the perpendicular from O on to PQ is:

$$a \sin \alpha - \tfrac{1}{3} a \sin 3\alpha.$$

Hence, OR being r and the angle AOR, θ, the polar equation of PQ is:

$$r \sin (3\alpha + \theta) = \tfrac{1}{3} a \sin 3\alpha - a \sin \alpha$$

or

$$r \sin (3\alpha + \theta) = -\tfrac{4}{3} a \sin^3\alpha \quad \text{.........................(i).}$$

Differentiating with respect to α,

we have:

$$r \cos (3\alpha + \theta) = -\tfrac{4}{3}a \sin^2\alpha \cos \alpha \quad \text{.........................(ii).}$$

The curve required is found by eliminating a between these two equations. By division we have

$$\tan(3a+\theta)=\tan a.$$

Whence
$$\theta=\pi-2a. \qquad \text{.................................(iii).}$$

And (i) becomes $r=\frac{2}{3}a(1+\cos\theta)$ the equation of a cardioid, cusp at O and passing through A.

The result (iii) shows that if R is the point of contact of PQ with the enveloping curve, then OR is parallel to CP. This suggests the following construction for the curve. A circle (Fig. 2) is graduated from A at intervals of 30°. The

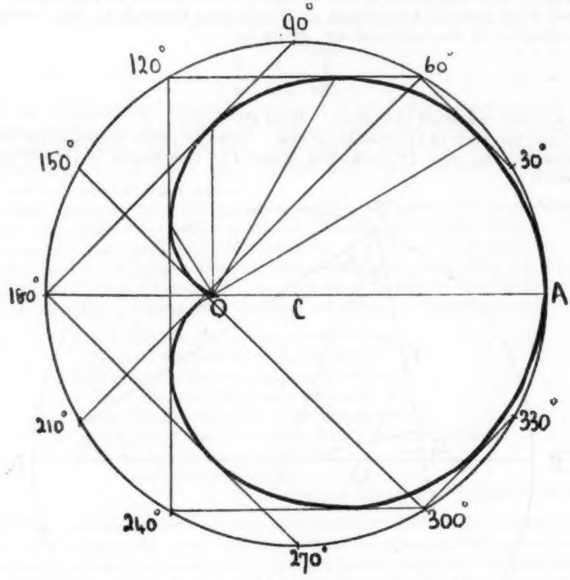

FIG. 2.

straight lines joining 30° to 60°, 60° to 120°, 90° to 180° . . . are the paths of rays reflected at the points 30°, 60°, 90°, . . . and straight lines through O parallel to C 30°, C 60°, C 90°, . . . meet these in their points of contact with the cardioid. We have thus 5 points and the tangents at them, from which the top half of the curve can be constructed. The lower half can be obtained similarly or by symmetry. If the circle had been divided at intervals of 15°, we should obtain 5 more points and tangents.

The fact that OR parallel to CP determines the point of contact of the reflected ray with the enveloping curve, can also be shown by Geometry. In Fig. 3, AP and AQ are consecutive rays, and PR, QR their reflected rays intersecting at R. QN is perpendicular to RP. It is obvious from elementary considerations that $Q\hat{R}P=3P\hat{A}Q$ and that $QCP=2QAP$. Hence $3PQ:CP=2QN:QR$. Hence $QN:PQ=3QR:2CP$. In Fig. 4, Q and P have reached coincidence, PQ has become the tangent at P and $QN:PQ$ has become $\cos RPC$ or $PV:2PC$.

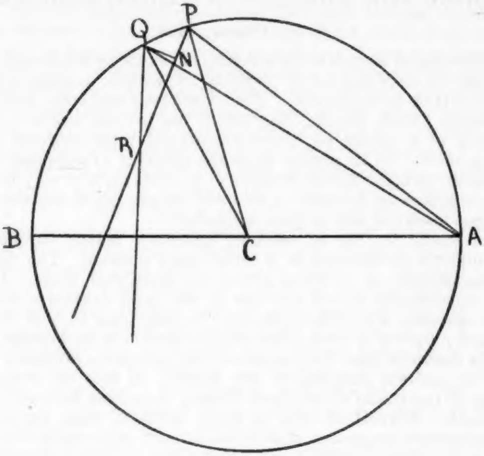

FIG. 3.

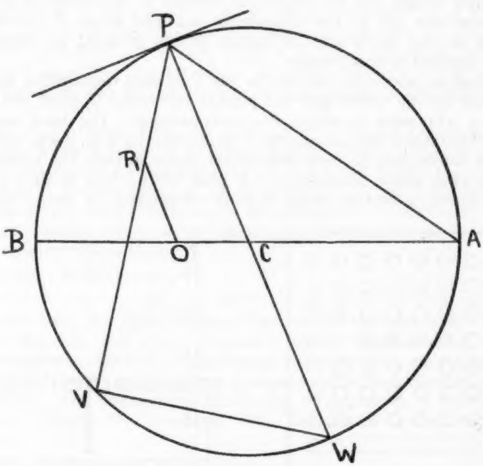

FIG. 4.

Hence $PV = 3PR$. R is of course the point of contact of PRV with the envelope, and RO parallel to PC meets CB in the fixed point O so that $CO = \frac{1}{3}CB$.

R.N. College, Dartmouth. H. E. PIGGOTT.

A MODEL FOR FIGURES IN THREE DIMENSIONS.

H. G. GREEN, M.A.

Now that the value of models in the teaching of Geometry has received emphasis in the *Report on the Teaching of Geometry in Schools*, a great impetus may be expected in their development. This expectancy combined with the article by Mr. Langley (*Math. Gazette*, May 1924), has led the writer to think that some account of a device for three-dimensional representation may be of interest to readers. In its present form the model is so arranged that figures for very general straight-line problems can be built up in a very few minutes. It has been in active use for about a year and has proved of valuable assistance both in trigonometrical and in pure geometry.

The ideal model for use in teaching geometry makes various demands which are difficult to combine in a satisfactory manner. The model which merely demonstrates an external shape has little real value. It must be capable of construction before the eyes of the pupil in such a way that the data of the question are each emphasised in due order so that the essential necessity and meaning of each piece of information is made clear. Further, it is strongly desirable that both algebraic and geometric processes of solution can be given concrete meaning on the model; in fact the setting up and examination of the model should lead directly to a close forecast of the steps in the solution. Beyond all this—a point which is often neglected in the design of laboratory apparatus, it is of the utmost importance that the model should contain nothing which is not vital to the problem in hand or likely to attract the attention of the pupil from it. While the author does not claim to have found a complete answer to these demands, he has found that its use in early stages has led to greater facility in the grasping of problems and has given new life to the apparently artificial steps of solution, representation of survey work and of formal proofs in solid geometry proving particularly fruitful in this respect.

The dimensions and other details in the following description are those of the one in use by the writer and can readily be varied to meet the particular needs of any who wish to adopt the arrangement. The base consists of a shallow box of outside measurements 11 in. square by 4 in. deep, made of $\frac{3}{8}$ in. wood; it is convenient to omit one of the sides so that the interior can be used for stowing away accessories. A plan of the box is shown in Fig. 1 and Fig. 2 shows a section with various accessories in use. The top has

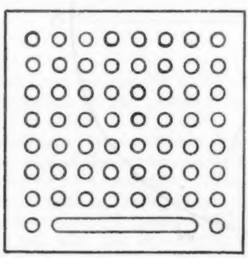

FIG. 1.

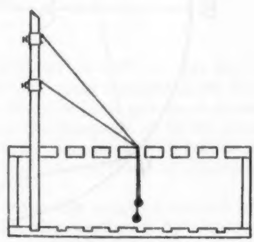

FIG. 2.

in it 7 rows of 8 holes each with an 8th row in which the 6 centre holes are replaced by a long slit; the centre of any hole is $1\frac{1}{4}$ in. from that of the adjacent one either in row or column, and they are made to a diameter of $\frac{1}{2}$ in., being then enlarged a little by means of a rat-tail file. Below each hole, on

the bottom of the box, is cut partially through the wood a small socket of like diameter. As it is important that the holes and sockets should be truly placed it is advisable that the pieces of wood which are to form the top and bottom should be temporarily screwed together so that a hole and its corresponding socket can be bored at the same time. Four or five masts, $\frac{1}{2}$ in. in diameter and 20 in. long, are needed (the cheap rods for light window curtains are very suitable), and also a number of clips which can be fastened at will on the masts. The type used on the writer's model consists of a brass tube having a tightening screw through a small brass block soldered to the tube, and on the other side a double-ended hook as illustrated on a section of mast in Fig. 3: certain types of fountain pen clip, however, serve quite well or, for a short time, a piece of wire wound tightly on the mast in the shape of a closed spring and then extended a little with the ends turned to form the hook. Cotton thread and about eight small lead balls or heavy beads of diameter approximately $\frac{3}{8}$ in., with loops to which the thread can be fastened, complete the essential part of the apparatus. In use the masts are held vertically by means of the holes and sockets, and a line in space can be formed by a thread from clip to clip or by a thread which has one end wrapped round a clip and the other supporting a ball through one of the holes. The top of the box should be painted or varnished in such a way that chalk marks giving projections of lines can be drawn on it.

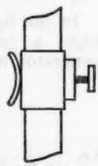

FIG. 3.

Individual needs dictate other additional pieces of apparatus: the writer has found particularly useful (1) a mast 8 in. long to represent a man or to mark without clumsiness an intermediate point on an inclined plane, (2) two wires forming coplanar arcs of circles of diameters 8 in. and 6 in., and having as a common base a clip tube 2 in. long, which, mounted on a mast through one of the holes at the end of the slit and lowered into it, can be used to illustrate the problems of "points where a flagstaff subtends given or maximum angles," and (3) a short pointer mounted stiffly on a clip tube so that it can be moved about an axis in the vertical plane to represent a sighting line.

To obtain the highest efficiency the use of a model demands some care and the point when it becomes no longer helpful must be closely watched for. It has been the custom of the writer to use it himself before the class in early stages, and in a further stage to allow the pupil to use it himself whenever he feels that it will help him towards the solution. It has been a matter of surprise how quickly many of the difficulties common to all beginners have been overcome by this means.

H. G. GREEN.

Univ. Coll., Nottingham.

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285. In my day, the most famous member [of the so-called "Apostles"] was Clark Maxwell, the great physicist, whose mathematical genius was already recognised. He was a fascinating object to me; propounding quaint paradoxes in a broad Scottish accent; capable of writing humorous lampoons upon the dons; and turning his knowledge of dynamics to account by contriving new varieties of "headers" into the Cam.—Leslie Stephen, *Early Impressions.*

286.

Elizabeth a Shakespeare own'd;
Charles could a Milton boast;
But Anne saw Newton high enthroned,
Amid the heavenly host. DIBDIN.

287. If the proof be demanded, I will not undertake to give it; the atoms of probability, of which my opinion has been formed, lie scattered over all his works. . . .—Johnson (on *Dryden*) 393, vol. ix. (Murphy).

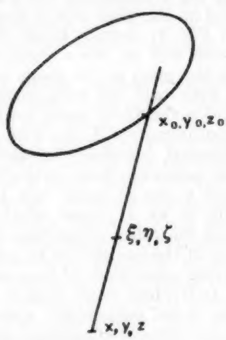
MATHEMATICAL NOTE.

749. [L[1]. 7. a.] *The Foci of a Conic.*

$$\frac{x}{a_1t^2+2b_1t+c_1}=\frac{y}{a_2t^2+2b_2t+c_2}=\frac{z}{a_3t^2+2b_3t+c_3}=\frac{1}{a_4t^2+2b_4t+c_4}.$$

The locus of the vertices of the right circular cones from which a given conic can be cut is a conic having the foci of the given section as vertices.

If then we can find this locus its intersections with the plane of the conic will give the foci required.



In the figure $(x_0y_0z_0)$ is a point on the conic, (xyz) a vertex, and $(\xi\eta\zeta)$ a point on the generator, and we have

$$\frac{\xi-x}{x_0-x}=\frac{\eta-y}{y_0-y}=\frac{\zeta-z}{z_0-z},$$

i.e. $$\frac{\xi-x}{(a_1t^2+2b_1t+c_1)-x(a_4t^2+2b_4t+c_4)}=\ldots=\ldots$$

or $$\frac{x'}{a_1't^2+2b_1't+c_1'}=\ldots=\ldots,$$

where $x'\equiv\xi-x,\quad a_1'=a_1-a_4x$, etc., etc.

The plane through (x, y, z) at right angles to this generator is

$$x'(a_1't^2+2b_1't+c_1')+y'(a_2't^2+2b_2t+c_2')+z'(a_3't^2+2b_3't'+c_3')=0,$$

and its envelope, the reciprocal cone, is

$$(a_1'x'+a_2'y'+a_3'z')(c_1'x'+c_2'y+c_3'z')=(b'_1x'+b_2'y'+b_3'z')^2,$$

or $$s_{11}'x'^2+s_{22}'y'^2+s_{33}'z'^2+2s_{23}'y'z'+2s_{31}'z'x'+2s_{12}'x'y'=0$$

where $$s_{\mu\nu}'=a_\mu'c_\nu'+a_\nu'c_\mu'-2b_\mu'b_\nu'.$$

The condition that this should be right circular is

$$s_{11}'-\frac{s_{31}'s_{12}'}{s_{23}'}=s_{22}'-\frac{s_{12}'s_{23}'}{s_{31}'}=s_{33}'-\frac{s_{23}'s_{31}'}{s_{12}'},$$

and this is the locus required.

Since (x, y, z) is to be in the plane of conic, we may write

$$x=\frac{a_1\lambda+b_1\mu+c_1\nu}{a_4\lambda+b_4\mu+c_4\nu},\ \text{etc.}$$

Hence

$$\begin{aligned}a_1'=a_1-a_4x&=\frac{a_1(a_4\lambda+b_4\mu+c_4\nu)-a_4(a_1\lambda+b_1\mu+c_1\nu)}{a_4\lambda+b_4\mu+c_4\nu}\\&=[(a_1b_4-a_4b_1)\mu-(c_1a_4-c_4a_1)\nu]/(a_4\lambda+b_4\mu+c_4\nu)\\&=(\gamma_1\mu-\beta_1\nu)/\rho,\ \text{say.}\end{aligned}$$

We have

$$\begin{aligned}(\beta_2\gamma_3-\beta_3\gamma_2)&=(c_2a_4-c_4a_2)(c_3b_4-a_4b_3)-(c_3a_4-c_4a_3)(a_2b_4-a_4b_2)\\&=a_4^2(b_2c_3-b_3c_2)+a_4b_4(c_2a_3-c_3a_2)+a_4c_4(a_2b_3-a_3b_2)\\&=a_4\begin{vmatrix}a_2&b_2&c_2\\a_3&b_3&c_3\\a_4&b_4&c_4\end{vmatrix}\equiv a_4\delta_1.\end{aligned}$$

Again, $A_1' \equiv b_2'c_3' - b_3'c_2'$

$$=(a_2\nu - \gamma_2\lambda)(\beta_3\lambda - a_3\mu) - (a_3\nu - \gamma_3\lambda)(\beta_2\lambda - a_2\mu)$$

$$= +\lambda^2(\beta_2\gamma_3 - \beta_3\gamma_2) + \lambda\mu(\gamma_2 a_3 - \gamma_3 a_2) + \lambda\nu(a_2\beta_3 - a_3\beta_2)$$

$$=\frac{\lambda}{\rho^2}(a_4\lambda + b_4\mu + c_4\nu)\delta_1 = \frac{\lambda}{\rho}\delta_1.$$

Finally $s_{31}'s_{12}' - s_{11}'s_{23}'$

$$=(\overline{a_1'c_3' + a_3'c_1'} - 2b_1'b_3')(\overline{a_1'c_2' + a_2'c_1'} - 2b_1'b_2')$$

$$-2(a_1'c_1' - b_1'^2)(\overline{a_2'c_3' + a_3'c_2'} - 2b_2'b_3')$$

$$=2A_2'C_3' + 2A_3'C_2' - B_2'B_3', \text{ which on reduction}$$

$$=(2\lambda\nu + 2\nu\lambda - \mu^2)\frac{1}{\rho^2}\delta_2\delta_3.$$

Hence the equations for the foci simplify to

$$\delta_1 s_{23}' = \delta_2 s_{31}' = \delta_3 s_{12}'. \qquad \text{(i)}$$

Again
$$s_{23}' = a_2'c_3' + a_3'c_2' - 2b_2'b_3'$$
$$=(a_2 - a_4 y)(c_3 - c_4 z)$$
$$+(a_3 - a_4 z)(c_2 - c_4 y)$$
$$-2(b_2 - b_4 y)(b_3 - b_4 z)$$
$$= s_{23} - s_{34}y - s_{24}z + s_{44}yz,$$

while for a parabola, $s_{44} = 0$.

The equations for the foci may be written

$$(s_{44}y - s_{24})(s_{44}z - s_{34}) - (s_{24}s_{34} - s_{23}s_{44}) = \frac{\lambda}{\delta_1}, \text{ etc.,}$$

if each of expressions (i) be called λ/s_{44} or

$$(s_{44}y - s_{24})(s_{44}z - s_{34}) = \frac{\lambda}{\delta_1} + f, \text{ say.}$$

Hence
$$s_{44}x - s_{14} = \pm\sqrt{\frac{\left(\frac{\lambda}{\delta_2} + g\right)\Big/\left(\frac{\lambda}{\delta_3} + h\right)}{\frac{\lambda}{\delta_1} + f}}, \qquad \text{(ii)}$$

Now
$$\delta_1 x + \delta_2 y + \delta_3 z + \delta_4 = 0;$$
and
$$\delta_1 s_{14} + \delta_2 s_{24} + \delta_3 s_{34} + \delta_4 s_{44} \equiv 0.$$

Hence multiplying the above equations (ii) in order by δ_1, δ_2, δ_3 and adding, we get

$$\frac{\delta_1}{\frac{\lambda}{\delta_1} + f} + \frac{\delta_2}{\frac{\lambda}{\delta_2} + g} + \frac{\delta_3}{\frac{\lambda}{\delta_3} + h} = 0. \qquad \text{(iii)}$$

This is a quadratic in λ, and the roots substituted in (ii) give the four foci. The axes are

$$(s_{44}x - s_{14})\left(\frac{\lambda}{\delta_1} + f\right) = \ldots = \ldots.$$

N. M. GIBBINS.

REVIEWS.

A School Mechanics. Part. I. By C. V. DURELL, M.A. Pp. xx+186. Answers i-x (at end). 3s. 6d. 1924. (G. Bell & Sons.)

Bitter experience, after trial to the contrary, has discovered that it is easier to begin the teaching of Mechanics with Statics, at any rate in teaching immature minds. Mr. Durell in this book is again venturing to begin with moving bodies, chiefly those moving with uniform acceleration, including projectiles, and even touching on angular velocity, and variable acceleration. In the old days, there was usually no experimental aid to this work, and it may be that with the kind of apparatus recommended by the author, young pupils may make better progress with this part of the subject. Mr. Durell is a teacher of great reputation, and no doubt his experience has been that experimental work with moving bodies stimulates the interest of pupils, and it may justify this revival of the ancient order of teaching.

The first three chapters all deal with the geometry of motion, kinematics. Statics, and the statical measure of force, including elastic strings and spiral springs, are introduced in Chapter IV., and moments are discussed in Chapter V. This is followed by a chapter on work and horse-power, connected with statical problems, including uniform motion; and the next chapter deals with the statics of machines, including pulley and cogwheel systems, screws and screw jack, and worm and wheel.

Then come chapters on kinetic energy and momentum, the equations for which are derived ingeniously from the previous kinematical work, *assuming* that a constant force produces constant acceleration. It is not till the last chapter [X., p. 171], that the formula $P/W=a/g$ is formally introduced. This inversion of the ordinary method of treatment is very remarkable, and does not seem to be quite logical.

The parallelogram and triangle of vectors are conspicuous by their absence, except in the preface, in which the author asserts that "the greater part of what is valuable in mechanics can be absorbed without the difficulties which accompany resolutions and composition." No doubt these will be treated in Part II., which is to complete the course up to Leaving Certificate and Matriculation requirements.

In the useful revision notes there seems to be a misleading suggestion (p. xvii) relating to power, the method given calculating only *average* power, the important formula Pv not being mentioned, though it is the formula which gives the actual rate of work at the moment. $\frac{Pv}{550}$ when P is in lb.-weight, and v in ft./second, or $\frac{Pv}{375}$ when v is in miles per hour, are worthy of mention in revision work, as giving horse-power exerted.

In the same notes, on p. xix, the last sentence in 10(v.) needs correction. One does not equate *forces* in calculating mechanical advantage, and it is not clear what is meant by "and then use (iv)."

P. 3, Example 8: "How far from Liverpool must the speed be raised to v knots," suggests that it is at that point that the speed is to be raised and that the vessel proceeds to New York at the speed v knots, whereas what is meant is that the steamer leaves Liverpool at v knots to make up for the x hours lost, and the question is how far must this speed be maintained to make up for the lost x hours. The sentence needs revision.

There are numerous examples of varying degrees of difficulty, well calculated to test and train the student. They are a very valuable part of the book.

There is one controversial point which is of considerable importance, viz., the effective horse-power with which a jet of water is driven through an orifice, which seems intimately connected with the distinction between actual H.P. and average H.P

If the jet is driven at v feet per second through an orifice of A square feet, the weight per second $=wAv$, its momentum per second is $\frac{wAv^2}{g}$, equal to the driving force P, and $\frac{Pv}{550}$ is the horse-power, $=\frac{wAv^3}{550g}$. Now, some writers,

including the author, take half this amount, basing it on the kinetic energy, but surely this is only the *average* horse-power. In fact, if we denote the constant quantity wAv by Q, $Q\dfrac{dv}{dt}$ is the resistance, and $\int Q\dfrac{dv}{dt}\,ds$, *i.e.* $\int_0 Qv\,dv$, is the work it can do after leaving the orifice. But this is with constantly diminishing velocity, and therefore can give only the *average* horse-power, when divided by 550.

The matter is important. The author in Ex. 17 on p. 142 asks for the horse-power of a certain jet on issuing from the orifice, and the answer given is 9·6, which is equal to its kinetic energy divided by 550. The answer should have been 19·2 if my contention is correct.

Prof. Loney, in his treatment of jets, and similar questions, always uses the formula Pv, which gives the result for which I am contending. The author's answer is in accordance wth his suggested method on p. xvii (previously alluded to) of calculating the horse-power exerted by an engine, which would certainly lead only to the average horse-power exerted during the "convenient interval of time" considered.

The book is well printed, well illustrated, and graphic methods of dealing with accelerated motion in a straight line are adequately given. Many experiments are carefully described and the results well discussed, and this should be of great assistance to teachers who wish to conduct such experiments.

A. Lodge.

An Introduction to the Mathematical Analysis of Statistics.

By C. H. Forsyth. Pp. viii + 241. 11s. 6d. net. (Chapman & Hall.)

Any book that takes a step in the direction of establishing statistical theory as a part of the regular undergraduate course for honours students is to be welcomed. The subject of statistics is young in comparison with many branches of the mathematical curriculum, but it is old enough to have grown into a system sufficiently well-knit for profitable study by students, and it provides a field rich in applications of many old-established topics of pure mathematics. It also introduces new concepts, such as frequency distribution and correlation, and it revivifies the maltreated subject of probability.

Whittaker and Robinson's *Calculus of Observations* is so comprehensive that it may well be supposed to render unnecessary any other book as a text for university students. Although it is a far less ambitious book, this by Professor C. H. Forsyth is in some respects superior to that of Whittaker and Robinson *as a text for the student.* The smaller book is far superior in its *Examples,* which are drawn from a very wide field and excellently illustrate the general principles of the subject. It is a pity that no Answers are supplied, for after the laborious calculations involved in many exercises of the statistical kind the student has a right to know whether he is right or not! On the other hand, some topics are treated so scantily (multiple correlation is an instance) that they are much more intelligible in the larger book.

There are few mathematical but there are many educational defects in the book. The treatment of the normal curve is mathematically unsatisfactory. Having shown that the curve must be (*a*) symmetrical about the vertical axis, (*b*) asymptotic to the horizontal axis, and (*c*) at a maximum where it crosses the vertical axis, the author proceeds: "All these properties of the curve can be expressed algebraically by the differential equation $dy/dx = -kxy$." It would surely have been better to arrive at this equation by a consideration of the probable mode of determination of the various values of the distributed variable (those values being determined by the operation of a large number of small causes of a "chance" nature). The proof of the equation of the normal curve given by Bertrand (and quoted in Whittaker and Robinson) is easy to follow, as also is the more usual and more clumsy method of deducing the equation by considering the limit of a binomial series.

The educational defects are mainly due to neglect of the principle that a student should not be required to study the means of solving problems of a certain kind before he has any idea of what those problems are. Thus, in

the first five chapters of this book the author is virtually preparing the way for the real subject of the book, and the student is handed in turn, for inspection and exercise, the weapons of Finite Differences, Gamma- and Beta-Functions, and Probability, without the least indication of the manner in which these weapons are subsequently to be used. The book, in fact, would be greatly improved if in subsequent editions the chapters were bound in the following order: VI., VIII., VII., X., I., II., III., IV., V., IX. Chapter VI. contains a most admirable account of Averages and Frequency Distributions, and would have made an excellent opening chapter. The chapter on Probability (Chap. V.) is fairly orthodox; it gives a useful discussion of the idea of a "homogeneous population," and might have been improved by a fuller account of empirical probability. The necessity for the tables of logarithms and antilogarithms at the end of the book is not apparent; surely every student using this book will have his own book of tables!

To the student who has acquired the art of skipping those parts of his text-books for which he feels no immediate need, the best advice is to study *The Calculus of Observations*, but even such a student will find this more elementary book suggestive and a good supplement to Whittaker and Robinson. There is no doubt room for a book, less advanced than the latter, and written by somebody who, being a teacher as well as a mathematician, realises that what may appear to the initiated as a "logical" order of presentation may not be the best order for teaching purposes. Need it be assumed, as it so often is, that order of presentation of material, creation of motive in the pupil for further study, choice of illustration calculated to appeal to the student, are matters that concern only schoolmasters, matters with which the teacher of university students need not be concerned?

E. R. Hamilton.

The Mathematical Groundwork of Economics. By A. L. Bowley, Sc.D., F.B.A. Pp. 98. 7s. net. 1924. (Clarendon Press.)

An elementary study of economics involves a certain amount of graphical or algebraical reasoning. Professor Bowley's book deals with the more advanced stages, and aims at coordinating the mathematical methods which have been developed in modern times by leading economists, particularly in relation to exchange, production, and taxation. The treatment is quite general, the reader being usually left to make his own applications. A good many graphs are given, and the reader's interest might have been increased if the facts represented by these graphs had been stated.

On some small points the notation is not entirely in accordance with the conventions of mathematicians. A more important matter is the apparent looseness of economists as to dimensions. Thus, if $V(x, y)$ is the utility enjoyed by A after having obtained x of "X" in exchange for y of "Y," the marginal utility of an increment of "X" is the partial differential coefficient of $V(x, y)$ with regard to x. But x is sometimes stated to be a quantity and sometimes a number: and $V(x, y)$ itself, which is a utility, is sometimes equated to a number. It is therefore uncertain whether utility and marginal utility are of the same or of different dimensions; and this uncertainty can hardly conduce to clear thinking.

W. F. S.

L'Espace, la Relation et la Position: essais sur le fondement de la géométrie. Par le Vicomte de Güell. Pp. 139. 10 fr. 1924 (Gauthier-Villars.)

The author is troubled in mind by the existence of postulates in geometry. He seeks to avoid them by defining a straight line as "a system of positions in a fixed direction," direction being accepted as indefinable, and parallels as "lines having the same relation of position to a transversal": which he imagines to contain the whole theory of parallels.

Clearer thinking is shown in some introductory remarks on the abstract nature of all geometries, and the practical value of Euclid's.

H. P. H.

Advanced Vector Analysis, *with Applications to Mathematical Physics*. C. E. WEATHERBURN. Pp. xvi. + 222. 15s. net. 1924. (Bell & Sons.)

This is a very useful and welcome supplement to the author's recent book on Elementary Vector Analysis.

It deals, in the first place, with the differential operators, and principally with the Hamiltonian ∇ as applied, scalarly or vectorially, to vector fields and scalar fields. The chapter closes with the orthogonal curvilinear expressions for the gradient, the divergence, curl, and the Laplacian, and with a good number of useful exercises. Chapter II. is devoted to line, surface and space integrals, leading naturally to the fundamental theorems of Gauss, Stokes, and Green; the freshman is also being sufficiently acquainted with Green's function for Laplace's equation, and the subject of this, as also of the following chapters, is rehearsed in a collection of problems and exercises. Chapters III. and IV. give the reader already the much desired applications to Physics, namely, the former to potential theory and heat conduction, and the latter to hydrodynamics, in absence of friction. The next chapter, however, brings us back to pure vector algebra (of course, with numerous illustrations), developing, that is, the theory of linear vector functions, and the corresponding operators, considered as dyads and dyadics, after Gibbs. This chapter contains also a clear presentation of the fundamental properties of quadrics, intimately associated with the said theory. The differentiation of dyadics and dyadics involving ∇ are then added, in Chapter VII., while the remaining Chapters VI., VIII., IX. and X. are all dedicated to physico-mathematical applications, their objects being rigid dynamics, statics of deformable bodies and dynamics of viscous fluids, the elementary theory of electricity and magnetism and, finally, Maxwell's and Lorentz's equations and the special-relativistic transformation.

The presentation is very clear and pleasant, yet concise enough. The book may be warmly recommended to mathematicians and to physicists as well.

LUDWIK SILBERSTEIN.

A Shorter School Geometry. Part I. By H. S. HALL and F. H. STEVENS. Pp. x + 164 + iv. 2s. 6d. 1924. (Macmillan.)

This book differs in plan and detail to a large extent from the well-known *School Geometry* by the same authors, and follows many of the suggestions made in the syllabus of Elementary Geometry put forward by the Incorporated Association of Assistant Masters in Secondary Schools, but it was prepared before the recent Report of the Mathematical Association appeared.

The introductory pages furnish a systematic course of experimental and practical geometry to precede the theoretical work which is developed in three sections. Section I. deals with Angles, Parallels, Congruent Triangles, Parallelograms; Section II. with Rectilinear Areas and Pythagoras' Theorem. Section III. with Simple Loci.

Most of the work is of that excellent quality which one expects from such well-known authors; but the treatment of parallels is inexcusably bad. Parallel straight lines are introduced as lines which have *like directions*, the *test* of like directions being the equality of corresponding angles made with a transversal. The authors conceal the fact that equal corresponding angles made with one transversal are not inconsistent with unequal corresponding angles made with another transversal, unless some appropriate parallel postulate is adopted. The result is a short road with a concealed turning,—so skilfully concealed that a non-Euclidean geometry appears unthinkable. The unsatisfactory character of such a treatment is effectively exposed in the recent Report issued by the Mathematical Association. The advantage of this method of treatment is that the sequence of the first few propositions becomes Euclid I. 13, 14, 15, 27, 29, 30, 32. To make this satisfactory it will be necessary to develop the properties of parallels in a different way, unless one is content to do ill that good (?) may come. W. J. DOBBS.

Osdoms. OSWESTRY EDUCATIONAL DOMINOES. (R. Platt, Ltd., Educational Publishers, Wigan.)

To guide the play-instinct of the young into channels educationally useful

is the aim of these ingenious games, devised by Mr. R. S. Williamson, B.A., Vice-Principal of the Cambridge University Training College, and Miss M. R. Williamson, of Oswestry Grammar School. The games provide the concrete means of expressing relationship. The child recognises pairs of symbols and in time remembers them by the mere force of repetition: the act of juxtaposition is the physical sign of apprehension of relationship. The act of rejection is also useful as the recognition of non-relationship.

The sets of games are as follows—some can be played by children of three or four, and all by those of five and upwards:

Colour. (i) Six colours. Match colour and colour. (ii) Eight colours. Match colour and name.

Geometrical. (i) Eight sunk shapes, training sense of touch (triangle, square, oblong, rhombus, rhomboid, circle, semicircle, ellipse). Match shape and shape. (ii) Same shapes but in outline. (iii) Same as (ii). Match shapes and name. (iv) Standard angles (30°, 45°, 60°, 90°, 120°, 135°, 150°).

Word Building. Three letter words. (i) Matching consonants with vowel followed by *d* or *g*, *e.g. b* and *-ad*, and (ii) Matching *b* or *c* followed by vowel with a consonant, *e.g. ca-* and *t*.

Arithmetical or Number. (i) Ten (graduated) sets deal with numbers up to 10 (addition and subtraction). An exhaustive and systematic course. (ii) Products 1 to 6. (iii) Products 6 to 10.

Another series deals with domestic animals and plants.

The advantages claimed are: cheapness (from 1s. 6d. a set); can be played without supervision by quite young children, and by a single child or group of children; variety; interest of material and method; practice in concentration of a simple kind; do not teach isolated facts but definite and consecutive units of knowledge . The proof of the pudding is in the eating. We shall be glad to receive accounts of the experience of those who have given these games a trial of reasonable duration.

Cambridge Readings in the Literature of Science: being extracts from the writings of Papers of Science to illustrate the development of Scientific Thought. Arranged by W. C. D WHETHAM and M. D. WHETHAM. Pp. xi+275. 7s. 7d. net. 1924. (Cambridge University Press.)

This is in many ways a most attractive compilation. Father and daughter have combined in the selection of characteristic and illustrative extracts, for the most part the *ipsissima verba* of those whose speculations and discoveries have contributed an effective impetus to the development of scientific thought, and here they present the reader with an Anthology of distinctive merit both literary and scientific.

"We have picked out as threads on which to string our anthology of science the ideas of mankind on three problems of transcending importance: (1) the structure of the universe—cosmogony; (2) the nature of matter—atomic theories; (3) the development of life—evolution." The passages selected under these three headings occupy respectively about 50, 100, and 100 pages.

From the familiar description of the Creation in the Book of Genesis, we pass to the speculations of the mighty Stagyrite, *il maestro di color che sanno*, placed by Dante above Socrates and Plato. The translation used is that of Thomas Taylor (1807). We learn why the heavens could not have been created, why the circle is the most perfect conceivable of all figures, why the universe revolves in a circle and itself is spherical, and why it is foolish to imagine that the earth has the shape of a drum.

For Aristarchus, the mathematician to whom we owe the concept of the heliocentric hypothesis, and for Archimedes, whose work "represents a sum of mathematical achievement unsurpassed by any one man in the world's history," we must, perforce, be taken to the pages of Sir Thomas Heath.

For Copernicus, the Canon of Frauenberg, we are led to that famous *De Revolutionibus Orbium Cœlestium*, which he just lived to see and to touch, and which was destined to bring headlong to the ground Aristotle's doctrine of a closed universe with the earth at its centre. We read in the *Sidereal Messenger* (Carlos's translation) the account given by "the Starry Galileo"

of his invention of the Telescope and his first observation. And so we pass from Newton to Laplace, to Foucault and Stokes and Bunsen and Kirchhoff, and the section closes with passages from Chap. VI.-XII. of Eddington's *Space, Time and Gravitation.* We could not have forgiven the compilers had they omitted the incomparable passages that close that wonderful book. Many readers will also be glad to be introduced in the next section to passages from the translation of Lucretius by Evelyn the Diarist, one of the members of the first Council of the Royal Society, and of so saintly a character as to win from the Prince of Dilettanti the title of "the Neighbour of the Gospel." On this translation Waller becomes quite dithyrambic :

> "For here *Lucretius* whole we find
> His words, his music, and his mind."

It is to be hoped that this anthology will be followed by others in due course.

1. **A Preparatory Arithmetic.** By C. Pendlebury. Fifth edition, revised and enlarged. Pp. xiv + 290 + xlviii. 3s. 1924. (Bell and Sons.)

2. **Revision Arithmetic and Mensuration.** By T. Thomas and J. J. P. Kent. Second edition. Pp. 128. 2s. net. 1924. (Mills and Boon.)

3. **A Shorter School Geometry.** Part II. By H. S. Hall and F. H. Stevens. Pp. viii + 165-304 + viii. 2s. 6d. 1924. (Macmillan.)

4. **Exercises in Trigonometry.** By E. R. Pigrome. Pp. 78. 1s. 6d. 1924. (Clarendon Press.)

5. **First Ideas of Trigonometry.** By B. A. Howard. Pp. 95. 1s. 9d. 1924. (Ginn and Co.)

(1) The first edition of this excellent text-book was published in 1912, close on the heels of the *Report on the Teaching of Arithmetic in Preparatory Schools* issued by our Association. The Syllabus of the Joint Committee of the Headmasters' Conference and the Association of Preparatory Schools have led to the new edition before us. In some instances the teacher will have to make a few cuts or to add material to bring it quite into line with our own last Report, but the alterations necessary are a small matter. The value of the book is largely increased by the sections on Mensuration. If we may be pardoned for making a suggestion to an author of the experience of Mr. Pendlebury, we would prefer a simpler treatment of such examples as No. 2 on p. 227 : *e.g.*

$$\sqrt{0\cdot144}=\sqrt{\frac{144}{10^3}}=\sqrt{\frac{1440}{10^4}}=\frac{\sqrt{1440}}{10^2} \text{ (and after } \sqrt{1440} \text{ is found)} = 0\cdot38 \text{ to two places.}$$

No explanation is then required. To some young minds "we mark off the periods from the decimal point both ways" is a piece of black magic. There are 101 Revision Papers of five questions each, and a few 40 min. Tests of 8 questions in each.

(2) Messrs. Thomas and Kent have in view Examinations for Army and Navy Entrance and for future engineers, and have produced a book that will be found quite useful for revision purposes. It is full of valuable hints and facts and has a useful section of half a dozen pages on the slide rule.

(3) This, the second and last part of the *Shorter Geometry*, contains three sections : IV. the Circle, V. Squares and Rectangles, VI. Proportion and Similar Figures. It includes the usual collection of riders and miscellaneous examples. It will "probably be found sufficiently comprehensive for an ordinary school course." The *Shorter Geometry* is marked by the well-known characteristics of the familiar *School Geometry* of the authors.

(4) The exercises here collected are for a two years' course. Most of the examples are designed to test methods rather than accuracy, and for them tables of ratios of angles in degrees only will suffice. The course covered extends to easy compound angles and solution of triangles. Answers are supplied. Diagrams are deliberately excluded, which may account to some extent for the low price of the book.

(5) An excellent introduction to the subject, "built up on three ratios instead of six." Examination claims necessitate the inclusion of the sections

on identities, "the only chapter in which the formal side of trigonometry is emphasised." We rejoice to see the early application to three-dimensional problems.

Graduated Problem Papers. By R. M. Wright. Pp. vii + 88. 5s. 6d. 1924. (Cambridge University Press.)

Mr. Wright's collection is for Scholarship Candidates, canditates for Part I. of the Mathematical Tripos, and for the Science Tripos, Part I. There are sixty papers carefully graduated, and in each paper are "included a certain number of questions which are little more than straightforward applications of well-known mathematical principles and methods." Answers are supplied where necessary. The collection is well adapted for its purpose.

Elementary Mathematical Problem Papers. By E. M. Radford. Pp. vi + 109. 4s. 1924. **Answers, with some Hints for Solutions.** Pp. 32. 2s. 6d. 1924. (Cambridge University Press.)

Many teachers will welcome "as filling a long-felt gap" such a set of elementary problem papers as Mr. Radford provides for Schools and Training Colleges. As weekly tests for those taking School Certificates, Matriculations, and Board of Education examinations, etc., they will be found invaluable, being largely original and skilfully graduated. The Answers, with Hints, are sold separately. The more advanced papers will break the ground for pupils who propose to attack the author's larger volume of more advanced problems.

ERRATA.

Gentle Reader; In a worke of this nature, it were strange to find no escapes at the presse; these few (onley) are observed as fit to be amended. (*Per* E. H. Neville. From Roe and Wingate: *Tabulae Logarithmicae*, 1633).

p. 93. Third line in § 4 should end with full stop.

p. 93, l. 25. *For* $\underset{\Delta x\to 0}{\mathrm{Lt}}\left(\frac{\Delta z}{\Delta x}\right)$ *read* $\underset{\Delta x\to 0}{\mathrm{Lt}}\left(\frac{\Delta z}{\Delta y}\right)$.

Footnote, lines 6 and 8 up. *For* $\underset{x\to b}{\mathrm{Lt}}\, f(y)$ *read* $\underset{y\to b}{\mathrm{Lt}}\, f(y)$.

p. 94, l. 5. *For* § 2 *read* § 3.

p. 249, l. 13. *For* were *read* rare.

CARDIFF BRANCH.

The Annual Meeting of the Branch was held on Oct. 20th. The officers were re-elected. The following is a year's programme as already drawn out:

Oct. 20th	Prof. Livens - - - -	Maxwell's Theory.
Nov. 3rd	Dr. Taylor - - - -	Non-Euclidean Geometry.
,, 17th	Mr. Pope, B.Sc. - - - -	Functions of a Complex Variable.
Dec. 1st	Dr. Dienes (Swansea Univ. Coll.)	Modern Geometry.

Next Term, Prof. Sutton-Pippard on "A Problem in Frameworks"; Miss Weighill on "Maps."

The number of student members has much increased.

Arthur T. Wadley,
Secretary.

A QUERY.

Can any reader tell the Librarian who was the author of *A Syllabus of the Differential and Integral Calculus, Part I.*, printed by R. Harwood, Bridge Street, Cambridge, 1825, and whether any other parts were published?

THE LIBRARY.

160 Castle Hill, Reading.

GIFTS.

Prof. **R. W. Genese**, who joined the A.I.G.T., in 1878, has made to the Library a notable gift, thus perpetuating a lifelong interest in the work of the Association.

(1) The following books:

J. C. Adams	Lunar Theory	1900
G. B. Airy	Errors of Observations {2}	1875
	Mathematical Tracts	1826
	Partial Differential Equations	1866
W. S. Aldis	Chapter on Fresnel's Theory of Double Refraction	1870
	Rigid Dynamics	1882
B. Aoust	Analyse Infinitésimale des Courbes Planes	1873
F. Auerbach	Taschenbuch für Mathematiker und Physiker	1909
F. Autenheimer	Differential- und Integralrechnung {3}	1887
P. Bachmann	Natur der Irrationalzahlen	1892
R. Baltzer	Déterminants Translated from German into French by J. Hoüel	1861
J. Bertrand	Traité d'Algèbre; II {: J. Bertrand et H. Garcet}	1878
	Calcul des Probabilités	1888
W. H. Besant	Roulettes and Glissettes	1870
E. Bézout	Cours de Mathématiques; III {: J. Garnier}	1797
J. B. Biot	Astronomie Physique (3 vols.) {2}	1810, 1811, 1811
J. Booth	New Geometrical Methods; I *The second volume is wanted*	1873
E. Brahy	Exercices Méthodiques de Calcul Intégral	1895
R. Burchett	Linear Perspective	1856
W. S. Burnside and A. W. Panton	Theory of Equations	1881
G. Cantor	Fondements de la Théorie des Ensembles Transfinis Translated from German into French by F. Marotte *The English translation of these epoch-making papers is already in the Library*	1899
J. Cape	Course of Mathematics (2 vols.) {5}	1857
F. S. Carey	Infinitesimal Calculus; II *Section I is not in the Library*	1918
L. N. M. Carnot	Métaphysique du Calcul Infinitésimal {2}	1813
A. L. Cauchy	Differenzialrechnung With J. B. J. Fourier's *Auflösung der bestimmten Gleichungen*, translated from French into German by C. H. Schnuse	1836
	Integralrechnung Translated from French into German by C. H. Schnuse	1846
E. Cesàro	Corso di Analisi Algebrica	1894
	Geometria Intrinseca	1896

R. F. A. Clebsch	Géométrie; I-II Edited by F. Lindemann; translated from German into French by A. Benoïst *In this copy the two volumes are bound together. The third volume, which deals with geometry of three dimensions, would be very welcome*	1879, 1880
A. Colas	Géométrie Elémentaire (2 vols.)	1885
L. Cremona	Projective Geometry Translated from Italian by C. Leudesdorf	1885
C. Culmann	Graphische Statik; I {2} ALL PUBLISHED OF THIS EDITION. Has 16 beautifully lithographed plates	1875
A. De Morgan	Explanation of Gnomonic Projection	1836
	Differential and Integral Calculus	1842
A. Desboves	Normales aux Coniques	1861
	Normales aux Surfaces du Second Ordre	1862
P. M. D'Ocagne	Coordonnées Parallèles et Axiales An eight-page section of this book was the beginning of nomography	1885
G. Dostor	Déterminants	1877
J. M. C. Duhamel	Calcul Infinitésimal; II: Calcul Intégral *An odd volume*	1861
	The same, another edition (2 vols.) {3: J. Bertrand}	1874, 1876
	Des Méthodes dans les Sciences de Raisonnement (5 parts) {2, 2, 1, 1, 1} Bound in two volumes	1875, 1878, 1868, 1870, 1873
J. D. Everett	Units and Physical Constants {2} The first edition (1875) had the title *Illustrations of the C.G.S. System of Units*	1879
F. Faà de Bruno	Théorie des Formes Binaires	1876
F. Frenet	Exercices sur le Calcul Infinitésimal {3 ( ) rep.}	1881
J. Frischauf	Absolute Geometrie nach Johann Bolyai	1872
P. Gilbert	Cours d'Analyse Infinitésimale {4}	1892
H. Goodwin	Course of Mathematics {4}	1853
	Problems and Examples adapted to the *Course* {2}	1851
	The same, another edition {3: T. G. Vyvyan}	1862
D. F. Gregory	Examples of Processes of the Calculus {2: W. Walton} This copy is interleaved and bound in two volumes	1846
J. Griffiths	Notes on the Geometry of the Plane Triangle	1867
J. A. Grunert	Lehrbuch der Mathematik für die mittlern Classen höherer Lehranstalten (2 parts) {2} Bound in one volume	1838, 1836
	Lehrbuch der Mathematik für die obern Classen höherer Lehranstalten (4 parts) {2} Bound in two volumes	1835-1836
C. Gudermann	Grundriss der analytischen Sphärik	1830
J. N. P. Hachette	Géométrie Descriptive (2 vols.) The second volume, without title-page, consists of the plates	1822
A. Harnack	Introduction to the Calculus Translated from German by G. L. Cathcart	1891

J. Hoüel	Essai critique sur les Principes Fondamentaux de la Géométrie Elémentaire {2} - - - - -	1883
P. Huber	Katechismus der Mechanik {3} - - - - -	1885
	The catechumenal method is seldom used at this level	
J. Hymers	Differential Equations and Finite Differences -	1839
	Theory of Equations {3} - - - - - -	1858
	Plane and Spherical Trigonometry {3} - - -	1847
F. Joachimsthal	Analytische Geometrie der Ebene {2} - - -	1871
E. Jouffret	Théorie de l'Energie - - - - - -	1883
J. Keily	Expansion of Structures by Heat - - -	1887
[Lord Kelvin]		
Sir W. Thomson [ ] and P. G. Tait		
	Elements of Natural Philosophy; I {2} - - -	1879
	All published	
S. F. Lacroix	Calcul Différentiel et Intégral - - - - -	1802
	Essais sur l'Enseignement {2} - - - - -	1816
C. A. Laisant	La Mathématique : Philosophie, Enseignement -	1898
	A presentation copy from the author to Prof. Genese	
V. Láska	Einführung in die Funktionentheorie - - -	1894
	Sphärische Trigonometrie - - - - - -	1890
V. A. Le Besgue	Introduction à la Théorie des Nombres - - -	1862
A. M. Legendre	Eléments de Géométrie {8} - - - - - -	1809
H. Lieber und F. von Lühmann		
	Geometrische Konstruktions-Aufgaben {6} - -	1882
R. O. S. Lipschitz	Untersuchungen ueber die Summen von Quadraten	1886
G. de Longchamps	Cours de Mathématiques Spéciales. I : Algèbre ; II : Géométrie analytique ; Supplément : Séries et calcul infinitésimal - - - 1883, 1884,	1885
	Bound in two volumes	
E. Lucas	Théorie des Nombres; I - - - - - -	1891
	All published	
J. Lüroth	Das Imaginäre in der Geometrie und das Rechnen mit Würfen - - - - - - - - -	1875
L. I. Magnus	Aufgaben und Lehrsätzen aus der analytischen Geometrie des Raumes; I - - - - - -	1837
A. Mannheim	Géométrie Descriptive {2} - - - - - -	1886
P. Mansion	Cours d'Analyse Infinitésimale - - - -	1887
H. C. E. Martus	Raumlehre; I - - - - - - - - -	1890
	Has any member the continuation to give ?	
A. Milinowski	Die gleichseitige Hyperbel - - - - - -	1883
F. N. M. Moigno	Mécanique Analytique : Statique - - - -	1868
J. Mulcahy	Modern Geometry {2} - - - - - - -	1862
	With chapters on spherical figures	
F. Müller	Zeittafeln zur Geschichte der Mathematik bis 1500	1892
R. Murphy	Algebraical Equations - - - - - - - -	1838
von Nagel	Geometrische Analysis. Anleitung zur Auflösung von Aufgaben auf reingeometrischem Wege {2} -	1876

J. Neuberg	Mémoire sur le Tétraèdre - - - - - - -	1884
	Systèmes de Tiges Articulées - - - - - -	1886
F. W. Newman	Difficulties of Elementary Geometry - - -	1841
B. Niewenglowski	Cours de Géométrie Analytique; III - - - *Two volumes are wanting*	1896
E. Ovazza e G. Sapegno		
	Calcolo Integrale - - - - - - - - - -	1883
A. J. N. Paque	Dissertation sur les vrais principes de l'Algèbre -	1863
M. Pasch	Einleitung in die Differential- und Integral-Rechnung	1882
G. Peano	Formulaire de Mathématiques; I-II - - Bound in one volume, with the introductory essay *Notations de Logique Mathématique* (1894). The inspiration of the work was Peano's, but in working out the details he had several collaborators	1895-1899
G. A. V. Peschka	Darstellende und projective Geometrie (8 vols.) Four volumes of text, each with an atlas of plates	1883-1885
P. C. J. Petersen	Equations Algébriques - - - - - - - - Translated from Danish into French by H. Laurent	1897
H. Picquet	Géométrie Analytique; I - - - - - - -	1882
	Systèmes ponctuels et tangentiels des Sections Coniques - - - - - - - - - -	1872
S. Pincherle	Analisi Algebrica - - - - - - - - -	1893
	Operazioni Distributive - - - - - - - - " In collaborazione con U. Amaldi "	1901
G. Pinna e G. Pedrazzi		
	Esercizi di Calcolo - - - - - - - - - Compiled from Peano's lectures and holographed	1883
S. D. Poisson	Mechanics (2 vols.) - - - - - - - - Translated from French and annotated by H. H. Harte	1842
P. G. D. de Pontécoulant		
	Théorie analytique du Système du Monde; II - *An odd volume*	1829
J. H. Pratt	Mathematical Principles of Mechanical Philosophy	1836
B. Price	Differential Calculus - - - - - - - - Not a part of the author's treatise in four volumes	1848
C. Pritchard	Theory of Couples - - - - - - - - -	1831
R. A. Proctor	The Cycloid and Cycloidal Curves - - - - -	1878
O. Pund	Algebra mit Einschluss der elementaren Zahlentheorie - - - - - - - - - - - (Schubert 6)	1899
L. Raffy	Applications Géométriques de l'Analyse - - -	1897
O. E. Randall	Descriptive Geometry - - - - - - - -	1905
	Shades and Shadows and Perspective - - -	1902
A. Rémond	Exercices de Géométrie Analytique; I - - - *Another odd volume*	1887
K. T. Reye	Geometrie der Lage; I-II - - - - -	1866, 1868
	Geometria di Posizione; I - - - - - - - Translated from German into Italian by A. Faifofer	1884
J. Richard	Géométrie Moderne - - - - - - - -	1898

R. P. Richardson and E. H. Landis	Fundamental Conceptions of Modern Mathematics	1916
L. Ripert	La Dualité et l'Homographie dans le Triangle et le Tétraèdre	1898
E. J. Routh	Rigid Dynamics	1860
C. Ruchonnet	Propriétés Générales des Courbes {4}	1880
J. W. Russell	Sequel to Elementary Geometry *Repairs a loss recorded a year ago*	1907
G. Salmon	Analytische Geometrie der Kegelschnitte {4} Translated into German by W. Fiedler	1878
J. A. Serret	Arithmétique {7 : J. A. Serret and C. de Comberousse}	1887
	Calcul Intégral {2} Fasc. 2 only	1880
W. Schell	Curven doppelter Krümmung	1859
O. Schlömilch	Compendium der höheren Analysis; I {2}	1861
	Übungsbuch zum Studium der höheren Analysis; II {3}	1882
	Geometrie des Masses Two parts in one volume	1854
L. Silberstein	Projective Vector Algebra	1919
J. C. Snowball	Plane and Spherical Trigonometry {8} A text-book, but a classic	1852
W. F. Stanley	Mathematical Drawing Instruments {5}	1878
O. Stolz	Allgemeine Arithmetik; I	1885
I. Stringham	Uniplanar Algebra	1893
P. G. Tait and W. J. Steele	Dynamics of a Particle {2}	1865
W. de Tannenberg	Applications Géométriques du Calcul Differentiel	1899
G. Tarry	Représentation Géométrique des Coniques et Quadriques Imaginaires	1886
G. Tomaselli	Esercizii sulle Equazioni Differenziali	1883
L. Trevigar	Sectionum Conicarum Elementa	1731
O. Veblen and J. W. Young	Projective Geometry; I *The second volume appeared in* 1918	1910
J. Vieille	Cours d'Analyse et de Mécanique Rationnelle	1851
W. Walton	Mechanical Problems	1842
G. D. E. Weyer	Neuere Konstruierende Geometrie	1891
W. Whewell	Conic Sections {3}	1855
	Doctrine of Limits	1838
	Dynamics (2 vols.) {3, 2}	1836, 1834
	Mechanical Euclid {3} " By calling this little work The Mechanical Euclid I mean to imply, that I have aimed at making it such a coherent system of exact reasoning, as that for which Euclid's name is become a synonym. Such a system of Mechanics, when once constructed, can hardly fail to be of use. . . ."	1838
	Mechanics {3}	1828
	First Principles of Mechanics	1832

B. WITZSCHEL Grundlinien der Neueren Geometrie - - - - 1858

J. P. WRAPSON and W. W. H. GEE
Mathematical and Physical Tables - - - - 1898

W. J. WRIGHT Invariants - - - - - - - - - - - - 1879
(Tracts relating to the Modern Higher Mathematics 3)

NOTE.—*The copy of Peano's* Formulaire *recorded above is imperfect, lacking pp.* 97-112 *of the first volume. If any reader of the* Gazette *had this part of the work from curiosity but did not maintain his interest in the production, there is a chance to render a very great service to the Association. When we obtain also the first part (pp.* 1-224) *of the fourth volume, our set of volumes* 1-5 *will be complete.*

(2) An anonymous treatise:

Lezioni di Analisi Infinitesimale - - - - - - - - 1873
Per gli allievi della Regia Accademia Militare
We have seen this book in a catalogue indexed under the author's name: G. Allievi!

A collection of *Notices sur les Travaux*, bound in one volume.
These are the accounts of their own work, submitted on application for admission to the Académie des Sciences, by Aoust, Appell, Halphen, Jordan, Laguerre, Mannheim, Picard, Poincaré

The two volumes of Mathematics in the *Library of Useful Knowledge* (1836, 1835).

Collections of examples by:

E. Bardey, R. Deakin, H. Dölp*, T. Gaskin*, O. Hermes, J. C. V. Hoffmann, A. Lonchampt*, T. Lund* (key to J. Wood's *Algebra*), H. McColl*, E. Mosnat*, G. Ritt, J. H. Smith, A. Wrigley.
* An asterisk indicates that the author supplies solutions or a commentary

Pamphlets and offprints by:

F. Casorati (2), W. Gallatly, R. W. Genese (3), O. Hesse, A. B. W. Kennedy, L. S. F. Koch, J. Neuberg (3), O. Rodrigues, O. Stolz, J. Taylor.

(3) PERIODICALS:

Cambridge and Dublin Mathematical Journal - - 1-3 1846-'48

L'Intermédiaire des Mathématiciens - - - - - 1-2, 4 1894-'95, '97

Journal de Mathématiques Elémentaires (Vuibert) - 1-12, 14-16
1877-'88, '90-'92

Journal de Mathématiques Elémentaires et Spéciales
(Bourget etc.) - - - - - - - - - - 5 1881
This journal was divided in the following year:

Journal de Mathématiques Elémentaires Sér. 2; 1-2, 4-5 1882-'83, '85-'86
Sér. 3; 1 1887

Journal de Mathématiques Spéciales - Sér. 2; 1-2, 4-5 1882-'83, '85-'86
Sér. 3; 1 1887
Actually the volumes for 1882 were inscribed Sér. 2, t. 6, but in the following year the number was Sér. 2, t. 2. It was presumably to avoid confusion that a third series was begun after Sér. 2, t. 5

Mathesis - - - - - - - 5-11 1885-'91
With these volumes are bound offprints by H. Brocard, B. I. Clasen, E. Gelin (3), A. Gob, E. Goedseels, E. Lemoine, W. M'Cay, P. Mansion (2), J. Neuberg (3), C. Le Paige, C. Le Paige and Fr. Deruyts, G. Tarry and J. Neuberg, C. Thiry, J. M. de Tilly (2), E. Vigarié (3) which were issued from time to time as supplements

Messenger of Mathematics - - - Ser. 2; 1-14 1872-'85
Also Nos. 2-4 of the first volume, 1862
If we could secure vol. 5 of the first series, our run would be complete from the foundation of the journal to 1885

Nouvelles Annales de Mathématiques	16	1857
	Sér. 2 ; 15-17	1876-'78
	Sér. 3 ; 7-10	1888-'91
Revue de Mathématiques Speciales -	1-3	1890-'96
Rivista di Matematica - - -	1-4	1891-'94

Only vol. IV is complete. Collation:—I; pp. 65-272. II; pp. 1-32. III; pp. 1-120

Also the Reports of the British Association for the years 1882, 1883, 1896-1918.

(4) School-books and text-books by:

W. M. Baker, R. S. Ball, C. W. C. Barlow and G. H. Bryan, W. H. Besant, H. Blackburn, J. Bonnycastle, W. Briggs and G. H. Bryan, C. Briot;

F. S. Carey, G. E. St. L. Carson and D. E. Smith (2, 4 vols.), J. Casey, E. Catalan, G. W. Caunt and C. M. Jessop, J. W. Colenso, S. Constable, F. Cuthbertson;

C. Davison, H. G. Day, A. De Morgan, W. J. Dobbs (2), N. F. Dupuis (2);

W. D. Eggar; N. M. Ferrers, W. S. Franklin B. MacNutt and R. L. Charles, H. Fleury;

J. A. Galbraith, J. A. Galbraith and S. Haughton (3, 2 vols.), W. Garnett (2), A. F. G. T. Gauss, T. G. Gribble, E. J. Gross;

H. S. Hall (2), F. H. von Hallerstein, G. B. Halsted, J. G. Hamilton and F. Kettle (2), J. Hann, S. H. Haslam and J. Edwards, H. St. J. Hunter;

L. Kambly (2 vols.);

R. Lachlan, R. Levett and C. Davison, H. B. Lübsen;

F. S. Macaulay, W. J. Macdonald, J. M'Dowell, F. G. Mehler (2 eds.), J. J. Milne and R. F. Davis, A. Mukhopadhyay; R. C. J. Nixon; M. O'Brien;

F. Reidt (2, 1 vol.), G. Richardson and A. S. Ramsey, R. A. Roberts, W. G. Ross, E. Rouché and C. de Comberousse;

J. Salomon, H. Schumann (5, 1 vol.), P. Scoones and L. Todd, C. Smith and S. Bryant, W. B. Smith;

C. Taylor, L. Tesar, I. Todhunter (4); C. Vacquant and A. M. de Lépinay;

J. Walmsley, H. W. Watson, B. Williamson (2), H. G. Willis, W. A. Willock, J. M. Wilson (2), J. Wolstenholme; K. E. Zetzsche.

The Librarian reports also the following gifts:

From Sir **George Greenhill**:

E. P. ADAMS and R. L. HIPPISLEY
Smithsonian Mathematical Formulae and Tables of Elliptic Functions - - - - - - - 1922
The tables of elliptic functions were compiled under the direction of the donor, who contributed an introduction to them

From Mr. **H. N. Haskell**:

Messenger of Mathematics, vol. 52 - - - 1922-'23

From Dr. **Hilda P. Hudson**:

An offprint by the late Miss G. D. Sadd, and
W. DE TANNENBERG Conférences sur les Transformations en Géométrie Plane - - - - - - - - - - - 1921

PURCHASES.

P. APPELL — Traité de Mécanique Rationnelle III - - - 1903
The Library still lacks the second volume of this work

A. L. CAUCHY — Bestimmte Integrale zwischen imaginären Grenzen 1900
(1825; Ostwald 112)

P. G. L. DIRICHLET Darstellung durch Sinus- und Cosinusreihen - - 1900
(1837; Ostwald 116)

P. DU BOIS REYMOND
Zwei Abhandlungen über ... Reihen - - - 1913
(1871, 1874; Ostwald 185)

C. F. GAUSS — Die im verkehrten Verhältnisse des Quadrats ... Kräfte - - - - - (1840; Ostwald 2) 1889

C. HUYGENS — Œuvres. Tomes 1, 8 - - - - - 1888, 1899
Odd volumes, bought because we have already five other volumes (4, 9, 10, 11, 14) of this definitive edition. Further donations are invited

P. E. B. JOURDAIN *On the general theory of functions* - - -
An offprint from Crelle. To make the collection of the author's mathematical papers complete, the Library needs only a few items

G. KIRCHHOFF — Mathematische Physik; I: Mechanik {3} - - 1883
The fourth volume is in the Godfrey collection, but two remain to challenge members' generosity

L. A. J. QUETELET *Sections coniques considérées dans le solide* - - 1821
This memoir presented to the Brussels Academy is an important addition to the collection of works on conics which the Association owes to the Rev. J. J. Milne

P. L. SEIDEL — Reihen welche discontinuirliche Functionen darstellen - - - - - (1847; Ostwald 116) 1900
This number of the *Klassiker* combines papers by Dirichlet and Seidel

Also a collection of publications of the International Commission on the Teaching of Mathematics, consisting of the reports which bear in the official list the following numbers:

GERMANY — Nos. 12-15, 17-20, 22-23, 25-27, 29-31, 34, 39-40, 43-45.

HUNGARY — No. 186.

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288. A professor of mathematics sent in the manuscript of a work on mechanics for the inspection of the Board, soliciting permission to publish it. . . . In describing the action of some mechanical apparatus the author stated that the wheels, springs, etc. worked *freely*; and further on he wrote that a straight line could be elongated into infinite space without the slightest *limit*; whereupon the censors struck out both words—the first without any comment, the second on the ground that the Russian emperor's authority was the only thing *without limit* in this world.—J. H. Urquhart, Trans. of Le Duc, *La Question Russe*, 1853.

289. (Mr. Joseph Ames) affected to understand the mathematics, and belonged to a society which assembled somewhere in Wapping; but his proficiency may be judged of, when, to my knowledge, he had no idea that two dissimilar bodies could have equal areas, namely, that a triangle could be equal to a square.—[He was secretary to the Society of Antiquaries, and was an F.R.S.] F. Grose, *The Olio*, 1796, p. 134.

PRINTED IN GREAT BRITAIN BY ROBERT MACLEHOSE AND CO. LTD
AT THE UNIVERSITY PRESS, GLASGOW

BOOKS RECEIVED, CONTENTS OF JOURNALS, ETC.

January, 1925.

Les lieux géométriques en mathématiques spéciales, avec application du principe de correspondance et de la théorie des caractéristiques à 1400 problèmes de lieux et d'enveloppes. By T. LEMOYNE. Pp. 150. 10 frs. 1923. (Vuibert.)

Cambridge Readings in the Literature of Science. Arranged by W. C. D. WHETHAM and M. D. WHETHAM. Pp. xi + 275. 7s. 6d. net. 1924. (Cam. Univ. Press.)

Hydrodynamics. By H. LAMB. 5th Edit. Pp. xvi + 687. 45s. net. 1924. (Cambridge University Press.)

Einstein's Theory of Relativity. By M. BORN. Pp. xi + 293. 12s. net. 1924. (Methuen.)

The Mathematical Theory of Relativity. By A. S. EDDINGTON. 2nd Edit. Pp. ix + 270. 20s. net. 1924. (Cam. Univ. Press.)

Linear Integral Equations. By W. V. LOVITT. Pp. xiii + 253. 15s. net. 1924. (McGraw-Hill, Bouverie St., E.C. 4.)

Œuvres de G. H. Halphen. Tome IV. Pp. v + 660. 100 fr. 1924. (Gauthier-Villars.)

Surveying for Schools and Scouts. By W. A. RICHARDSON. 2nd Edit. Pp. 110. 2s. 6d. 1924. (G. Philip & Sons.)

The Mathematical Groundwork of Economics. An Introductory Treatise. By A. L. BOWLEY. Pp. viii + 98. 7s. net. 1924. (Clarendon Press.)

First Ideas of Trigonometry. By B. A. HOWARD. Pp. 95. 1s. 9d. 1924. (Ginn.)

Binomial Factorisations, giving Extensive Congruence Tables and Factorisation Tables. Vol. II. Pp. 4 + lxxix + 215. 15s. *Binomial Factorisations.* Vol. VI. (Supplement to Vol. II.) Pp. 3 + 103. 5s. By Lieut. Col. ALLAN J. C. CUNNINGHAM. 1924. (F. Hodgson.)

An Elementary Course of Analytical Geometry. By B. C. MOLONEY. Pp. viii + 167 + xvi. Part I. 2s. 6d. Complete, 3s. 6d. 1924. (Bell.)

Elements of the Theory of Infinite Processes. By LL. L. SMAIL. Pp. viii + 339. 17s. 6d. net. 1924. (M'Graw-Hill, 6 Bouverie St., E.C. 4.)

On the Problem of Four Particles. By J. BRILL. Pp. 4. Reprint from *Proc. Cam. Phil. Soc.* xxii. 2.

On the Direct Numerical Calculation of Elliptic Functions and Integrals. By L. V. KING. Pp. viii + 42. 3s. 6d. net. 1924. (Cam. Univ. Press.)

The Theory of Relativity. Studies and Contributions. By A. HENDERSON, A. W. HOBBS, and J. W. LASLEY, Jr. Pp. xiii + 99. 11s. 6d. net. 1924. (Per Ox. Univ. Press.)

A School Mechanics. By C. V. DURELL. Pp. xiv + 186 + x. 3s. 6d. 1924. (Bell.)

Journal of the Royal Society of Arts. May 23, 1924. *The Free Pendulum.* By F. HOPE-JONES, M.I.E.E. Pp. 446-460.

Orders of Infinity. The "Infinitärcalcül" of P. du Bois-Reymond. By G. H. HARDY. 2nd Edit. Pp. 77. 6s. net. 1924. (Cam. Univ. Press.)

Statique et Résistance des Matériaux. By PAUL MONTEL. Pp. 275. 30 fr. 1924. (Gauthier-Villars.)

Creación de la Estación experimental de Hidráulica y de Electrotécnica : Anuario para era año 1903. Published by the National University of La Plata.

The Growth of Legend about Sir Isaac Newton. Pp. 390-392. By F. CAJORI. (*Science*, May 2, 1924.)

Elementary Mathematical Problem Papers. By E. M. RADFORD. Pp. 109. 4s. *Answers, with some Hints for Solution*, 2s. 6d. 1924. (Cambridge Univ. Press.)

Mathematics for Technical Students. By E. R. VERITY. Pp. xi + 468. 12s. 6d. net. 1924. (Longmans, Green.)

American Journal of Mathematics. (Johns Hopkins Press, Baltimore.)
April, 1924.

Two-dimensional Tensor Analysis without Coordinates. Pp. 71-94. G. Y. RAINICH. *Fractional Operations as applied to a Class of Volterra Integral Equations.* Pp. 95-109. H. T. DAVIS. *Representation of Three-element Algebras.* Pp. 110-116. B. A. BERNSTEIN. *The Riemann Adjoints of Completely Integrable Systems of Partial Differential Equations.* Pp. 117-130. C. A. NELSON. *Further Types of Involutorial Transformations which leave each Cubic Surface of a Web Invariant.* Pp. 131-141. V. SNYDER.

July, 1924.

Geometric Aspects of the Abelian Modular Functions of Genus Four. Pp. 143-192. A. B. COBLE. *The Curve of Ambience.* Pp. 193-201. FRANK MORLEY. *On a Class of Invariant Subgroups of the Conformal and Projective Groups in Function Space.* Pp. 201-214. I. A. BARNETT.

Oct. 1924.

A General Class of Problems in Approximations. Pp. 215-234. D. JACKSON. *On a Rational Plane Quintic Curve into Four Real Cusps.* Pp. 235-240. P. FIELD. *Projection Properties of a Ruled Surface in the Neighbourhood of a Ruling.* Pp. 241-257. A. F. CARPENTER. *Complete Characterization of Dynamical Trajectories in N-space.* Pp. 258-272. L. M. KELLS. *Rods of Constant or Variable Circular Cross-Section.* Pp. 273-287. C. A. GARABEDIAN.

The American Mathematical Monthly. (Lancaster, Pa.)
Sept. 1924.

Integral Inequalities with Applications to the Calculus of Variations. Pp. 326-337. O. DUNKEL. *A System of Triangles related to a Poristic System.* Pp. 337-340. J. H. WEAVER. *Geometrical Construction of points on a 4-leaf rose.* Pp. 340. H. H. DOWNING. *Note on a solution of a set of linear equations.* Pp. 340-341. J. P. BALLANTINE. *Evaluation of a determinant* $|1/(r+s-1)!|$. Pp. 341-342. J. J. NASSAU. *On the division of a circumference into 5 equal parts.* P. 342. H. C. BRADLEY. *On the definition of determinants.* Pp. 343-345. A. A. BENNETT. *The nature of the Correlation-Coefficient.* P. 346. M. H. ROESER.

Oct. 1924.

A Theorem on Isogonal Tetrahedra. Pp. 371-375. B. H. BROWN. *Uniqueness of the Lorentz Transformation.* Pp. 376-382. A. CHURCH. *Infinite and Imaginary Elements in Algebra and Geometry.* Pp. 383-387. R. M. WINGER. *Early History of Division by Zero.* Pp. 387-389. H. G. ROMIG. *Dominical Letters and Perpetual Calendars.* Pp. 389-392. W. K. NELSON. *Some Results involving* π. Pp. 392-394. R. S. UNDERWOOD.

Annals of Mathematics. (Princeton, N.J.)
March 1924.

On the method of least mth powers for a set of simultaneous equations. Pp. 185-192. D. JACKSON. *The derivative of the general integral.* Pp. 193-204. P. J. DANIELL. *On a class of transformations in function space.* Pp. 205-220. L. A. BARNETT. *Congruences of circles studied with reference to the surface of centres.* Pp. 221-237. J. M. THOMAS. *On Neumann's existence proof.* Pp. 238-240. W. F. OSGOOD. *On the automorphic functions of the group* (0, 3 ; 2, 4, 6). Pp. 241-260. H. B. DALAKER. *Note on Dirichlet's Series with complex exponents.* Pp. 261-278. E. HILLE. *On types of monoidal involutions.* Pp. 279-284. V. SNYDER. *An inequality for the roots of an algebraic equation.* Pp. 285-286. J. L. WALSH.

Bolletino della Unione Matematica Italiana. (Zanichelli, Bologna.)
Oct. 1924.

Sui teoremi del potenziale elastico. Pp. 145-150. G. SUPINO. *Sul secondo principio di reciprocità.* Pp. 150-154. P. BONANNO. *Sulla ricersa del punti doppi apparenti di una quartica di pirma specie.* Pp. 154-156. E. ZONDADARI. *Sopra alcune serie di polinomi di Legendre.* Pp. 156-159. C. MINEO. *Sulla determinazione del saggio d'interesse nelle rendite costanti.* Pp. 160-161. L. LORDI. *Sopra un teorema di Noether.* Pp. 162-167. C. ROSATI.

Bulletin of the American Mathematical Society. (Lancaster, Pa.)
Oct. 1924.

Invariance of the Poincaré Numbers of a Discrete Group. Pp. 405-406. O. VEBLEN. *Analytic Functions and Periodicity .* Pp. 406-409. J. F. RITT. *A Convergence Proof for Simple and Multiple Fourier Series.* Pp. 410-416. M. G. CARMAN. *On certain Topics in the Mathematical Theory of Statistics.* Pp. 417-453. H. L. RIETZ.

The Eugenics Review. (Macmillan.)
Jan., May, Oct. 1924.

L'Intermédiaire des Mathématiciens. (Gauthier-Villars.)
Nos. 7-10, July-Sept. 1924.

The Journal of the Indian Mathematical Society. (Varadachari, Madras.)

April, 1924.

Supplement to Vol. XV.: Report of the Fourth Conference of the Society. (Varadachari, Madras.)

June 1924.

Invariant-Factors and Integer-Sequences. Pp. 189-198. R. VAIDYANATHASWAMY. *Twin Points.* Pp. 199-210. M. B. RAO. *Definitions of Straight Line.* Pp. 211-212 *s.v.* S. V. RAMAMURTHY. *Notes and Questions.* Pp. 138-152.

August, 1924.

A Classical Indian Puzzle-Problem. Pp. 214-223. A. A. KRISHNASWAMI AYYANGAR. *Summable Double Series.* Pp. 224-232. V. TIRUVENKATACHARYA. *Some Infinite Products and Series.* Part II. Pp. 233-236. By M. B. RAO and M. VENKATARAMA AYYAR. Notes :—*Did the Greeks solve the Quadratic Equation?* Pp. 153-155. G. A. MILLER. *On the Eight Spheres Touching the Faces of a Tetrahedron.* Pp. 155-157. C. SULDANHA and B. B. BAGI.

Periodico di Matematiche. (Zanichelli, Bologna.)

July, 1924.

I principali trattati di Algebra dalle origini della stampa al 1800. Pp. 277-306. A. AGOSTINI. *L'Equazione di quarto grado. Risoluzione di L. Ferrari e sua interpretazione geometrica.* Pp. 327-334. V. NOTARI. *Numeri decimali periodici e loro generatrici.* Pp. 335-339. A. PRIOLO.

Nov. 1924.

La struttura dell' atomo e le proprietà magnetiche. Pp. 397-407. P. LANGEVIN. *I fondamenti della teoria degli insiemi di Cantor.* Pp. 408- . O. ZARISKI. *Di un presunto errore di logica di Cauchy.* Pp. 438-440. F. SIBIRANI.

Proceedings of the Edinburgh Mathematical Society. (Bell & Sons.)

XLII. Part I. (1923-4.)

The Numerical Evaluation of Double Integrals. Pp. 2-13. A. C. AITKEN and G. L. FREWEN. *The Equation of Telegraphy.* Pp. 14-45. Miss M. C. GRAY. *The Linear Complex belonging to the Invariant System of Three Quadrics.* Pp. 29-40. J. WILLIAMSON. *Geometrical Interpretation of a few Concomitants of the Cubic in the Argand Plane.* Pp. 41-45. W. SADDLER. *On the Numerical Solution of Integral Equations.* Pp. 46-59. G. PRASAD.

April, 1924.

The Numerical Evaluation of Double Integrals. Pp. 2-13. A. C. AITKEN and G. L. FREWEN. *The Equation of Telegraphy.* Pp. 14-28. C. GRAY. *The Linear Complexes belonging to the Invariant System of Three Quadrics.* Pp. 29-40. J. WILLIAMSON. *Geometrical Interpretation of a few Concomitants of the Cubic in the Argand Plane.* Pp. 41-45. W. SADDLER. *On the Numerical Solution of Integral Equations.* Pp. 46-59. G. PRASAD.

Proceedings of the Physico-Mathematical Society of Japan. (Tôkyô Imperial University.)

July, 1923.

On the Mean Modulus of a Polynomial. Pp. 77-89. S. KAKEYA.

Sept. 1923.

Oct.-Dec. 1923.

The Atomistic Mechanism of Metal Rolling. Pp. 150-168. U. KAKINUMA.

Feb. 1924.

March, 1924.

Some Relaticist Problems of Rotation. Pp. 14-27. T. KATSUCHI.

Revista de Matematicas y Fisicas Elementales. (Buenos Aires, 44 Bulnes.)

Feb. 1924.

March, 1924.

Sumaciones varias de términos en progresión geométrica. Pp. 209-212. TR. A. DORDEA. *Observacion sobre un problema de minima.* Pp. 212-214. P. SERGESCU.

April, 1924.

Revista Matemática Hispano-Americana. (Soc. Mat. Española, Madrid.)

March 1924.

Análisis Situs Combinatorio. Pp. 33-41. H. WEYL. *Sobre la traslación paralela infinitesimal.* Pp. 42-49. F. PEÑA. *Resultante de Bezout.* Pp. 50-53. J. R. SANZ.

April, 1924.

Generalización de un teorema de Jacobi. Pp. 65-80. H. GERMAY. *Sobre la Traslación paralela infinitesimal.* Pp. 81-89. F. PEÑA.

May, 1924.

Don Z. G. de Galeano. Pp. 97-103. *Sobre la Traslación Paralela Infinitesimal.* Pp. 104-110. F. PEÑA. *Acerca de la Enseñanza da las Matemáticas en las Escuelas Industriales.* Pp. 117-122. T. M. ESCOBAR. *Nomograma para resolver la "Ecuación de Segundo Grado."* Pp. 123-124. A. SALDAÑA.

June, 1924.

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